Three Types of Producer's and Consumer's Risks in the Single Sampling Plan

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The classical producer's risk and the classical consumer's risk are defined in acceptance sampling based on the assumption that the proportion defective of incoming lots is a constant. This assumption has been a focus of much of the criticism of acceptance sampling in recent years. In this paper, we assume that the proportion defective is a random variable that follows a beta distribution, and we derive the modified producer's risk and the modified consumer's risk. We also derive the Bayes producer's risk and the Bayes consumer's risk. In addition, we clarify the relationships of the modified and Bayes risks with the classical risks.

Introduction

SINGLE sampling plans for attributes are characterized by the lot size, N; the sample size, n; and the acceptance number, c. Let X be the number of defective items observed in the random sample of n units inspected. In this paper, we assume that the inspection is perfect, even though errors are inevitable in any inspection process (Johnson, Kotz, and Wu (1991)). If X is less than or equal to c, the lot will be accepted; otherwise, the lot will be rejected. Thus, where a single sampling plan is to be adopted, the first decision should be the determination of n and c for a given N.

A common approach to the design of a single sampling plan is based on the producer's risk, α, and the consumer's risk, β. Traditionally, α is defined as the probability of rejecting a lot in which the proportion defective, p, of the lot is the same as the producer's acceptable quality level (AQL):

$$\alpha = \Pr\left[X > c \mid p = \text{AQL}\right]. \tag{1}$$

Similarly, β is defined as the probability of accepting a lot in which the p of the lot equals the lot tolerance proportion defective (LTPD):

$$\beta = \Pr\left[X \le c \mid p = \text{LTPD}\right]. \tag{2}$$

The definitions of α and β are based on the assumption that incoming lots are formed from a production process that is stable with a constant p.

The concept of a production process that is in stable condition and has a fixed proportion of defective items plays a fundamental role in acceptance sampling. For such a production process, the probability of producing a defective is a constant p, and the defective items occur at random. Thus, if p and n are given, X can be fully determined. However, the assumption of the constant p has been the target of much of the criticism of acceptance sampling. For example, Mood (1943) proved that if a controlled process is known to produce a fixed proportion of defective items, then the number of defective items found in a random sample is independent of the number of defective items in the remainder of the population.

Using Mood's theorem, Deming (1986) and others have condemned the use of acceptance sampling for processes that are stable because the independence theorem renders any inference about the number of defective items in the population that is made based on the analysis of a sample of those items invalid. Gitlow et al. (1995 p. 442) claimed that "... as processes are stabilized as a result of quality efforts, acceptance plans that are valid for chaotic processes—albeit at high cost—will no longer be effective on the stable process." Vardeman (1986, p. 327) ar-

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