General

On the Information Economics Approach to the Generalized Game Show Problem

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The game show problem has received considerable attention since it appeared in Marilyn vos Savant's column in the *Parade Magazine*. I consider in this article the player's optimal decision strategy and the probability of winning the prize in a generalized version of the game show problem. By means of the information economics approach, we can easily (1) represent the host's various strategies as a simple matrix form; (2) extend the problem to more-than-three-door cases; (3) incorporate the player's prior information; and (4) consider the problem in which the player's choice behavior depends not on the probability of winning the prize, but on the expected utility in the generalized game show problem. The intricacies of this wonderfully confusing little problem make it an excellent tool in teaching probability and statistics courses.

KEY WORDS: Bayes rule; Decision analysis; Information economics; Probability models.

A simple probability problem has provoked a series of disputes between a columnist said to be the holder of the world's highest IQ score and numerous Ph.D.s in academia since it appeared in the "Ask Marilyn" column in the September 9, 1990, issue of *Parade Magazine*. The original three-door game show problem appeared in the column is stated as follows: "Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, 'Do you want to pick door No. 2?' Is it to your advantage to switch your choice?—Craig F. Whitaker, Columbia, MD."

Ms. Marilyn vos Savant, listed in the *Guinness Book of World Records Hall of Fame* for highest IQ in the world, replied "Yes; you should switch. The first door has a one-third chance of winning, but the second door has a two-thirds chance." When she innocently printed the reply in the magazine supplement to many Sunday newspapers, she had no idea that it would provoke a national controversy. She received thousands of letters, nearly all insisting that, because two options remained, the chances were even. The most vehement criticism has come from statisticians and scientists, who have alternated between gloating at her and lamenting the nation's innumeracy. As she received more letters, she defended her original claim repeatedly in the December 2, 1990, issue, February 17, 1991, issue, and July 7, 1991, issue of the same magazine.

The debate raging among statisticians, readers of *Parade Magazine*, and fans of the television game show, "Let's Make a Deal," was also reported in the July 21, 1991, issue of the *New York Times*. Recently, the game show problem was aired in Tom and Ray's *CarTalk* radio show in October 25, 1997, and since then they reportedly have received "satchels full of mail." They decided to put the puzzle and its solution on the CarTalk Web page, http://cartalk.com/about/monty, saying "what could be more fun than proving a bunch of pompous academics wrong!!"

Note that the same problem with different names had been discussed in academic journals before it appeared in Ms. vos Savant's column in 1990. Selvin (1975), for example, considered the probability of winning the prize behind one of three curtains in the TV game show hosted by Mr. Monty Hall. He called it the Monty Hall Problem in *The American Statistician* and concluded that switching doubles the chances of winning the prize from 1/3 to 2/3. Nalebuff (1987) also proposed the TV game show problem as a puzzle in *Economic Perspectives* and gave the same answer. Later, Kotz and Stroup (1983) and Siegel (1990) used the puzzle for instructional purposes in their introductory statistics books.

After the game show problem appeared in the vos Savant's column, several attempts to end the debate were also reported in academic journals and newsletters. Based on a decision tree with conditional probabilities, for example, Chun (1991) illustrated in *OR/MS Today* that it is to the player's advantage to take the switch. In subsequent issues of the newsletter, the editor published a representative sample of the letters he received, saying that "no subject has inspired more letters to the editor of *OR/MS Today* than the Game Show Problem."

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