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A NONPARAMETRIC CONTROL CHART FOR A SYMMETRIC PROCESS: A MARKOVIAN APPROACH

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Introduction

Statistical process control charts have been widely used as a fundamental tool of modern quality management. In most process control charts, a decision-maker plots sample points in order of occurrence on a graph that has upper and lower control limits. The *control limits* are usually chosen so that if there are no special causes of variation affecting the process, nearly all of the sample points would fall between the upper and lower control limits. Thus, unless there is any point that plots outside of the control limits, the decision-maker concludes that the process is in a state of statistical control. Thus, the determination of the control limits, along with the sample size and the sampling frequency, is one of the most important tasks in designing a control chart.

In most cases, the control limits are determined based on the basic assumption that the individual observations—or, at least, the subsample means—follow a *Normal* distribution. In many practical situations, however, a decision-maker may have sufficient reason to doubt the validity of the normality assumption. Based on an empirical study of a sample of 235 quality control applications, for example, Alwan and Roberts (1) concluded that "violations of assumptions are the rule rather than the exception in practice."

One type of violation is that the underlying distribution has heavier tails than the Normal distribution, as shown in Figure 1a. We can also imagine a situation where a process is skewed to the right, as shown in Figure 1b. In some situations, the distribution may have more than one mode, as shown in Figure 1c.

The normality assumption is not only critical in the design of control charts for variables but also has significant effects on the evaluation and selection of a control chart. The performance of a control chart is usually measured by the average run length (ARL), and when the data do not follow the assumed Normal distribution, the calculated ARL may be significantly different from the actual ARL.

Due to the importance of the normality assumption in the design and evaluation of control charts, several authors have investigated the effect of non-normality on such control charts as the $\overline{X}$ and R charts. Burr (2), for example, concluded that "$\overline{X}$ and R charts are quite robust relative to non-normality" and "we can use the ordinary normal curve control chart constants unless the population is markedly non-normal." Schilling and Nelson (3) have also studied the effect of non-normality on the control limits of the $\overline{X}$ chart and concluded that, in most cases, samples of size four or