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Viewpoints

Economic optimization of off-line inspection procedures with inspection errors

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Recently, a colleague and I embarked on a research project involving the design of off-line inspection procedures with directional information. During our literature search, we found an article in *Journal of the Operational Research Society* that dealt with our area of interest. The Sheu *et al*'s (2003) article extended Raz *et al*'s (2000) original off-line inspection model by considering inspection errors. After a month of work, we were finally able to replicate their numeric results, but in the process found significant algebraic errors in the article. It is in the interest of other researchers that may in the future use the information presented in the article that I bring these errors to your attention.

First, consider a sequence of n items that have been produced from the same production line with a constant failure rate, $1-p$. Each item is either 'defective' or 'non-defective'. In the presence of inspection errors, they introduced an indicator variable X_j in page 890 to represent the inspection result of the jth item as follows:

$$X_j = \begin{cases} 1 & \text{if the } j\text{th item is tested non-defective} \\ 0 & \text{if the } j\text{th item is tested defective} \end{cases}$$

However, we need to define another binary variable Y_j to represent the 'true' quality of the jth item in the sequence:

$$Y_j = \begin{cases} 1 & \text{if the } j\text{th item is non-defective} \\ 0 & \text{if the } j\text{th item is defective} \end{cases}$$

The algebraic errors in their article are due to their indifference between the *observed* inspection result X_j and the *unobservable* state Y_j of the jth item. Specifically, they misused $\mathrm{P}[Y_j|Y_n]$ in place of $\mathrm{P}[Y_j|X_n]$ in the optimal 'no-inspection policy' in page 891. Note that those two conditional probabilities are equal if and only if there are no inspection errors.

Suppose that the last item in the sequence of n items is tested defective (ie $X_n=0$). Then, the conditional probability that the jth item is 'true' non-defective is

$$\begin{aligned}
&P[Y_j=1|X_n=0] \\
&\quad = P[Y_j=1|Y_n=1]P[Y_n=1|X_n=0] \\
&\qquad + P[Y_j=1|Y_n=0]P[Y_n=0|X_n=0] \\
&\quad = P[Y_j=1|Y_n=1]\frac{p^n\alpha}{p^n\alpha+(1-p^n)(1-\beta)} \\
&\qquad + P[Y_j=1|Y_n=0]\frac{(1-p^n)(1-\beta)}{p^n\alpha+(1-p^n)(1-\beta)} \\
&\quad = \frac{p^n\alpha}{p^n\alpha+(1-p^n)(1-\beta)} \\
&\qquad + \frac{(p^j-p^n)}{(1-p^n)}\frac{(1-p^n)(1-\beta)}{p^n\alpha+(1-p^n)(1-\beta)} \\
&\quad = \frac{p^n\alpha+(p^j-p^n)(1-\beta)}{p^n\alpha+(1-p^n)(1-\beta)}
\end{aligned}$$

where α and β are Type I and Type II errors, respectively.

Likewise, the conditional probability that the jth item is 'true' defective when the last item is tested defective is

$$\begin{aligned}
&P[Y_j=0|X_n=0] \\
&\quad = P[Y_j=0|Y_n=1]P[Y_n=1|X_n=0] \\
&\qquad + P[Y_j=0|Y_n=0]P[Y_n=0|X_n=0] \\
&\quad = P[Y_j=0|Y_n=0]\frac{(1-p^n)(1-\beta)}{p^n\alpha+(1-p^n)(1-\beta)} \\
&\quad = \frac{(1-p^j)}{(1-p^n)}\frac{(1-p^n)(1-\beta)}{[p^n\alpha+(1-p^n)(1-\beta)]} \\
&\quad = \frac{(1-p^j)(1-\beta)}{p^n\alpha+(1-p^n)(1-\beta)}
\end{aligned}$$

Consequently, the equation in page 891 after Equation (5) in Sheu *et al* (2003) should be written as

$$\frac{(1-p^j)(1-\beta)}{p^n\alpha+(1-p^n)(1-\beta)}c_p = \frac{p^n\alpha+(p^j-p^n)(1-\beta)}{p^n\alpha+(1-p^n)(1-\beta)}c_s$$

where c_p and c_s are misclassification costs. This equation correctly considers the inspection errors α and β in the no-inspection policy, while their original equation is independent of the inspection errors.

Furthermore, the break-even point in Equation (6) should be

$$j' = \frac{1}{\log p}\log\frac{(1-\beta)c_p + p^n(1-\beta)c_s - p^n\alpha c_s}{(1-\beta)(c_p+c_s)}$$