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Bayesian Analysis of the Sequential Inspection Plan via the Gibbs Sampler

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A complex product, such as a software system, is often inspected more than once in a sequential manner to further improve its quality and reliability. In such a case, a particularly important task is to accurately estimate the number of errors still remaining in the product after a series of multiple inspections. In the paper, we first develop a maximum likelihood method of estimating both the number of undiscovered errors in the product and the detection probability. We then compare its performance with that of an existing estimation method that has several limitations. We also propose a Bayesian method with noninformative priors, which performs very well in a Monte Carlo simulation study. As the prior knowledge is elicited and incorporated in the analysis, the prediction accuracy of the Bayesian method improves even further. Thus, it would be worthwhile to use various estimation methods and compare their estimates in a specific inspection environment.

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1. Introduction

Suppose that a certain complex product, such as an automobile, a mobile home, or a software system, has an unknown number N of defects, errors, faults, or nonconformities. Because of inspection errors, the product will be inspected more than once in a sequential manner to further improve its quality and reliability. After a series of k inspection cycles, the inspection results will be represented as a set $\mathbf{x} = \{x_1, x_2, \ldots, x_k\}$, where x_i is the numbers of defects discovered and removed during the ith inspection cycle. In the paper, we consider the problem of estimating, based solely on the inspection results $\mathbf{x}$, (i) the number of undetected errors (or, equivalently, the total number of errors N initially contained in the product) and (ii) the inspector's unknown detection probability p.

Estimating the number of errors still remaining in the product is an important task in reliability engineering. In software reliability, for example, the accurate estimation of the number of undetected errors "not only helps certify the application-readiness of the software, but also provides an indication of the effort that will be needed for customer support and for the upgrading of future program releases" (Jewell 1985a, p. 663). A similar interpretation arises in proof-reading a manuscript for typographical errors, inspecting a new home for construction flaws (Bonett and Woodward 1994), or screening a production lot for quality assessment and assurance.

Estimating the inspection effectiveness is another important task in quality management. For example, the airport security authority is interested not only in the number of illegal items not detected during the inspection process, but also in the detection probability of the inspection procedure. Based on the inspection effectiveness, we may determine the number of inspections required to achieve a desired level of product quality (Greenberg and Stokes 1995).

The *sequential* inspection model considered in the paper should be contrasted with the *parallel* inspection model, in which several inspectors are put to work independently on identical copies of the product, secretly identifying, but not removing, the defects that they find. In such a case, some defects discovered by one inspector could be also detected by other inspectors. In the *sequential* inspection plan, on the other hand, any defects discovered during an inspection cycle are removed or corrected so that they will not leak through the subsequent inspection cycles. Thus, the same defects will not be discovered more than once in the sequential inspection process.

Note that the sequential inspection plan can be classified further into the continuous-time case and the discrete-time case. In the *continuous-time* case, faults are discovered one after another in random order, and the inter-discovery time between two consecutive faults is assumed to be a continuous random variable, usually described as a nonhomogeneous Poisson process. In the software reliability growth model, for example, the inspection history is expressed as a set $\mathbf{t} = \{t_1, t_2, \ldots, t_k\}$, in which t_i is the time between the discoveries of the $(i-1)$th and ith faults. In the discrete-time version of the sequential inspection model, the number of faults x_i detected during the ith inspection cycle is revealed at the end of the discrete time period. Thus, the inspection results are summarized as a