IIE Transactions (2010) **42**, 304–316

ISSN: 0740-817X print / 1545-8830 online
DOI: 10.1080/07408170903228975

Bayesian inspection model for the production process subject to a random failure

YOUNG H. CHUN

Department of Information Systems & Decision Sciences, E. J. Ourso College of Business Administration, Louisiana State University, Baton Rouge, LA 70803-6316, USA
E-mail: chun@lsu.edu

Received January 2009 and accepted July 2009

Consider a sequence of items produced on a high-speed mass production line which is subject to a random failure. When an item in the sequence is inspected it is possible to obtain directional information about the exact timing of a process failure—before or after producing the inspected item. Using this directional information this paper proposes Bayesian inspection procedures that deal with three related problems: (i) how often to inspect items on the production line; (ii) how to conduct the search for more defective items; and (iii) when to stop the search process and salvage the remaining items. Based on various cost factors, the problem of optimal inspection interval, optimal search process and an optimal stopping rule is formulated as a profit-maximization model via a dynamic programming approach. For the production process with an unknown failure rate, Bayesian methods of estimating the process failure rate are proposed. The proposed Bayesian inspection procedures can be applied to a wide variety of high-speed mass production processes such as printing labels, filling containers or mixing ingredients.

Keywords: Inspection, search model, Bayesian estimation, renewal-reward process, dynamic programming

1. Introduction

Consider a mass production facility for printing labels, filling containers, or mixing ingredients. At the beginning of a production run, the facility starts "in control" after initial setups and only conforming items are produced. The production process is subject to a random failure and, once the process goes "out of control," it starts to produce non-conforming items. During the production process, we inspect every nth produced item. We call this a "regular" inspection. Once we find a non-conforming item, we stop the production process and start "special" inspections to detect more non-conforming items in the last sequence of n items.

In such a situation, the first problem of interest is how often to conduct a *regular* inspection. The inspection interval is usually determined by: (i) the inspection cost which is usually known; and (ii) the process failure rate which can be estimated from past failure data. The second problem of interest is how to conduct the *special* inspections. The last sequence of n items is composed of non-defective items, followed by defective items. We need to derive the optimal search procedure that economically identifies more non-defective items and detect more defective items from the sequence. If the cost of inspection is relatively high, we may stop the search process early and salvage all the remaining items in the sequence with no further inspections. Thus, the third problem of interest is when to stop the special inspection process.

In this paper, we propose an inspection procedure that simultaneously determines "how often to inspect" items on the production line, "how to search" for more defective items and "when to stop" the search process and salvage the remaining items. The optimal inspection procedure is derived based on several cost factors such as the market value of a non-defective item, salvage value of a discarded item and inspection cost of an item.

Because we inspect every nth item and stop the production process as soon as we find a defective item, the production process can be described as a renewal-reward process (Ross, 1983). In this article, we formulate the renewal-reward process as a profit-maximization model via a stochastic dynamic programming approach. As in Hassin's classic paper (Hassin, 1984), we confine our attention to a special case in which the production process is subject to a random shock and the process failure rate is constant. We also propose several statistical methods to estimate the process failure rate and compare their performances in numerical analyses. As expected, the empirical Bayes estimate with a beta prior performs very well when historical data is available.