LSU E. J. OURSO COLLEGE OF BUSINESS

Dr. Chun's Numb3rs & L∅gic

Game Show Problem

Young H. Chun, Ph.D.

***Professor of Decision Science* &**
Cherie H. Flores Endowed Chair in MBA Studies

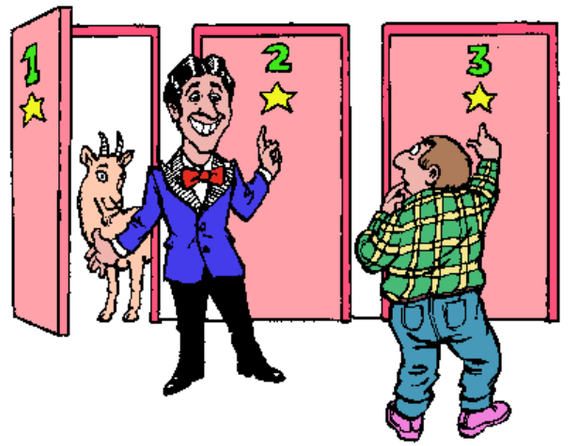
Game Show Problem

Ask Marilyn, *Parade Magazine*, (Sep. 9, 1990)

"Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, the others, goats.

You pick a door, say #1, and the host, who knows what's behind the doors, opens another door, say #3, which has a goat. He says to you, 'Do you want to pick door #2?'

It is to your advantage to switch your choice of doors?"



Craig F. Whitaker, Columbia, Maryland.

* Background (aka Monty Hall Problem)

Ms. Marilyn vos Savant, listed in the "Guinness Book of World Records Hall of Fame" for "Highest IQ" in the world, replied that, "Yes; you should switch. The first door has a one-third chance of winning, but the second door has a two-thirds chance."

When she innocently printed the reply in the magazine supplement to many Sunday newspapers, she had no idea that it would provoke a national controversy. She received thousands of letters, nearly all insisting that, because two options remained, the chances were even.

The most vehement criticism has come from statisticians and scientists, who have alternated between gloating at her and lamenting the nation's innumeracy.

Whose side are you on?

- Google search?

A. Letters from Academia

- Since you seem to enjoy coming straight to the point, I'll do the same. In the following question and answer, you blew it! As a professional mathematician, I'm very concerned with the general public's lack of mathematical skills. Please help by confessing your error and it the future being more careful.

  Robert Sachs, Ph. D. George Mason Univ.

- You blew it, and you blew it big! ... There is enough mathematical illiteracy in this country, and we don't need the holder of the world's highest I.Q. propagating more. Shame!

  S. S., Ph.D. University of Florida

- Your answer to the question is in error. But if it is any consolation, many of my academic colleagues have also been stumped by this problem.

  Barry Pasternack, Ph. D. California Faculty Association

- Your logic is in error, and I am sure you will receive many letters on this topic from high school and college students. Perhaps you should keep a few addresses for help with future columns.

  W. Robert Smith, Ph.D. Georgia State University

- You're in error, but Albert Einstein earned a dearer place in the hearts of people after he admitted his errors.

Frank Rose, Ph.D. University of Michigan

- I have been a faithful reader of your column, and I have not, until now, had any reason to doubt you. However, in this matter (for which I do have expertise), your answer is clearly at odds with the truth.

James Rauff, Ph. D. Millikin University

- May I suggest that you obtain and refer to a standard textbook on probability before you try to answer a question of this type again?

Charles Reid, Ph.D. University of Florida

- You are utterly incorrect about the game show question, and I hope this controversy will call some public attention to the serious national crisis in mathematical education. If you can admit your error, you will have contributed constructively towards the solution of a deplorable situation. How many irate mathematicians are needed to get you to change your mind?

E. Ray Bobo, Ph.D. Georgetown University

- I am in shock that after being corrected by at least three mathematicians, you still do not see your mistake.

Kent Ford, Dickinson State University

- You are wrong, but look at the positive side. If all those Ph.D.'s were wrong, the country would be in some very serious trouble.

  Everett Harman, Ph.D. U.S. Army Research Institute

- You are the goat!

  Glenn Calkins, Ph.D. Western State College

B. Tom and Ray's *Car Talk* Radio Show

- "What could be more fun than proving a bunch of pompous academics wrong!!!"

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B. Tom and Ray's *Car Talk* Radio Show

- "What could be more fun than proving a bunch of pompous academics wrong!!!"

C. Who is your Daddy?

- **Young H. Chun**, "On the Information Economics Approach to the Generalized Game Show Problem," *The American Statistician*, 53, (1999), pp. 43-51.

- **Young H. Chun**, "Game Show Problem," *OR/MS Today* (June, **1991**), p. 9.

Door #1	Door #2

- **You choose Door #1**

P[Car?]	1/3	1/3	~~1/3~~

- **He opened Door #3**

P[Car? | Open #3]	1/3	1/3	=> No way!
P[Car? | Open #3]	1/3	2/3	=> IQ
P[Car? | Open #3]	1/2	1/2	=> Ph.D.

- Envelopes, please!

- **Why so much controversy**

  over such a simple problem?

- **You are the judge…**

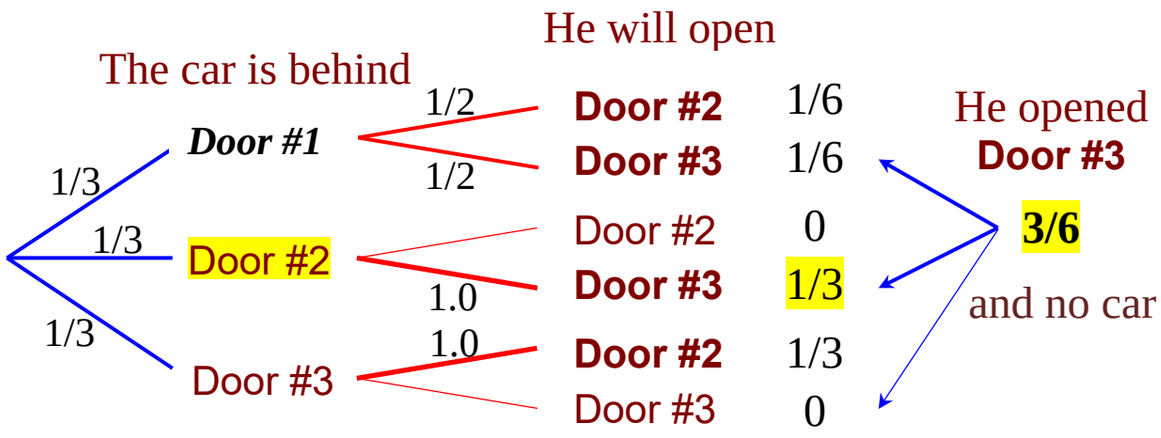
- **Whose side are you on?**

	#1	#2	#3		
P[Car? | Open #3]	1/2	1/2	0	=>	**Ph.D.**
P[Car? | Open #3]	1/3	2/3	0	=>	**I.Q.**

- More envelopes, please!

* Solution #1: You always have a second chance!

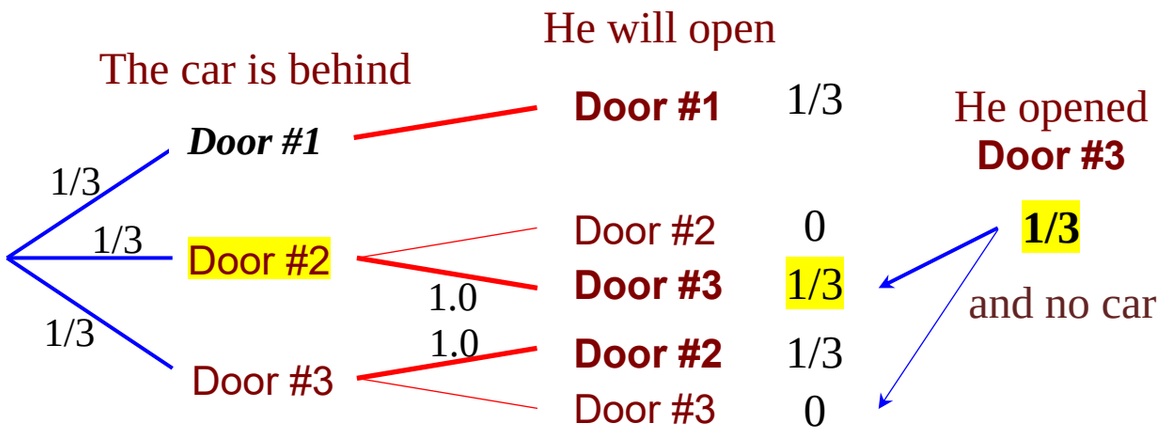
You chose *Door #1*		He will open		
		Door#1	Door#2	Door#3
If the car is behind	Door#1		1/2	1/2
	Door#2			1
	Door#3		1	



- P[Car #1 | Opened #3] = 1/6 out of 3/6 = **1/3**
- P[Car #2 | Opened #3] = 1/3 out of 3/6 = **2/3** **Switch to #2!**

* Solution #2: The game show host is your friend!

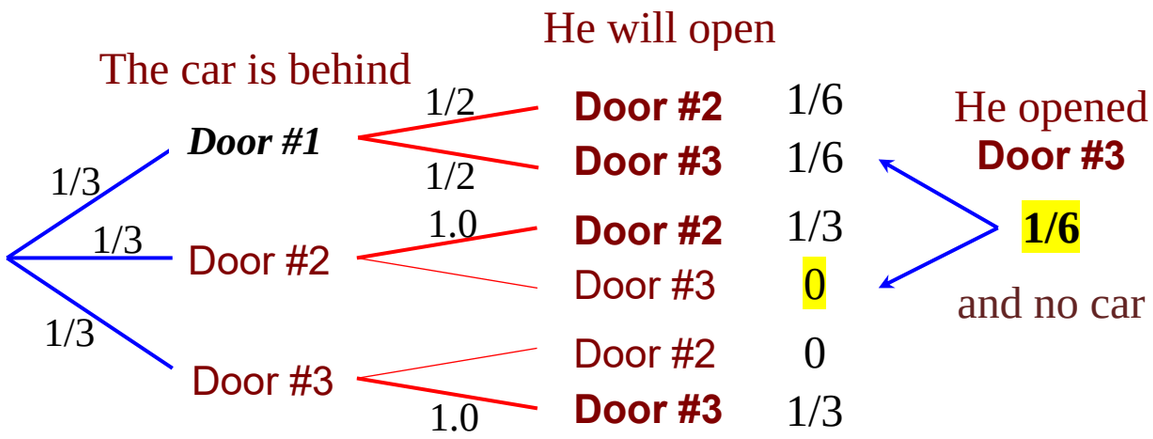
You chose Door #1		He will open		
		Door#1	Door#2	Door#3
If the car is behind	Door#1	1		
	Door#2			1
	Door#3		1	



- P[Car #1 | Opened #3] = 0 out of 1/3 = **0**
- P[Car #2 | Opened #3] = 1/3 out of 1/3 = **1.0** **Switch to #2!**

* Solution #3: The game show host is your enemy!

You chose *Door #1*		He will open		
		Door#1	Door#2	Door#3
If the car is behind	Door#1		1/2	1/2
	Door#2		1	
	Door#3			1



- P[Car #1 | Opened #3] = 1/6 out of 1/6 = **1.0**
- P[Car #2 | Opened #3] = 0 out of 3/6 = **0.0** **Don't Switch!!**

* Solution #4: No idea if he is your friend or enemy!

(i.e., No idea if you always have the second chance.)

- P[Car #1 | Opened #3] = **1/2**
- P[Car #2 | Opened #3] = **1/2**

Switch or not, it doesn't matter!

Original Letter in Ask Marilyn, *Parade Magazine*, (Sep. 9, 1990)

"Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, the others, goats.

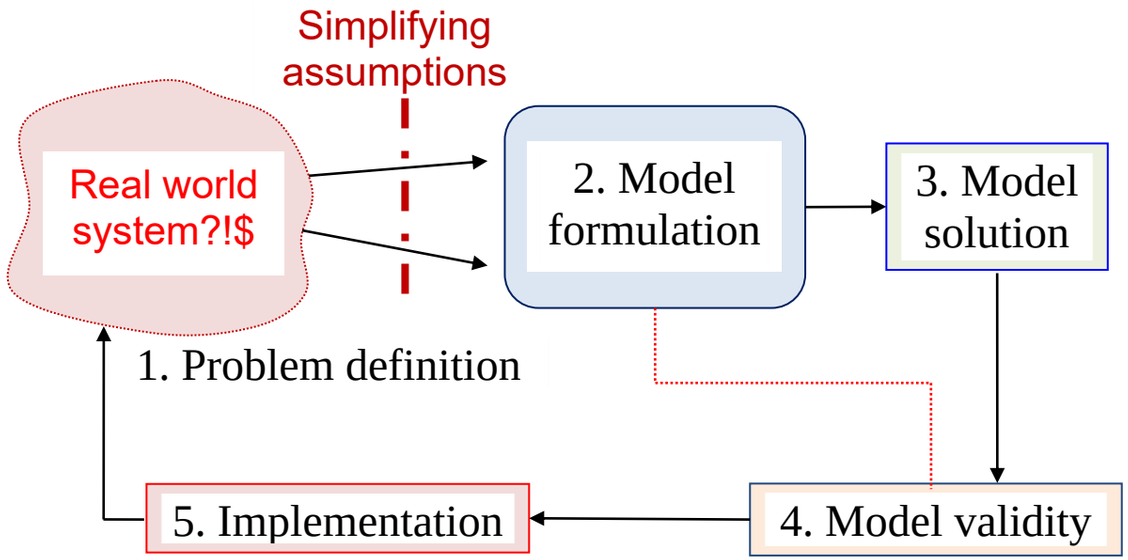
You pick a door, say #1, and the host, who knows what's behind the doors, opens another door, say #3, which has a goat. He says to you, 'Do you want to pick door #2?'

It is to your advantage to switch your choice of doors?"

Craig F. Whitaker, Columbia, Maryland.

Are you always given the second chance?

* Mathematical Modeling



Ex] *Linear* regression model: $Y = bX$

(Mileage = 11,000 x *Age* of your car)

* Game Show Problem

- GIGO: Garbage in, Garbage out!
- Assumption is the mother of all screw-ups

The **rule of the game** is not clear… => **Final Verdict!**

* Variation: The game show host is drunk!

(i.e., The game show host has no idea where the car is...)

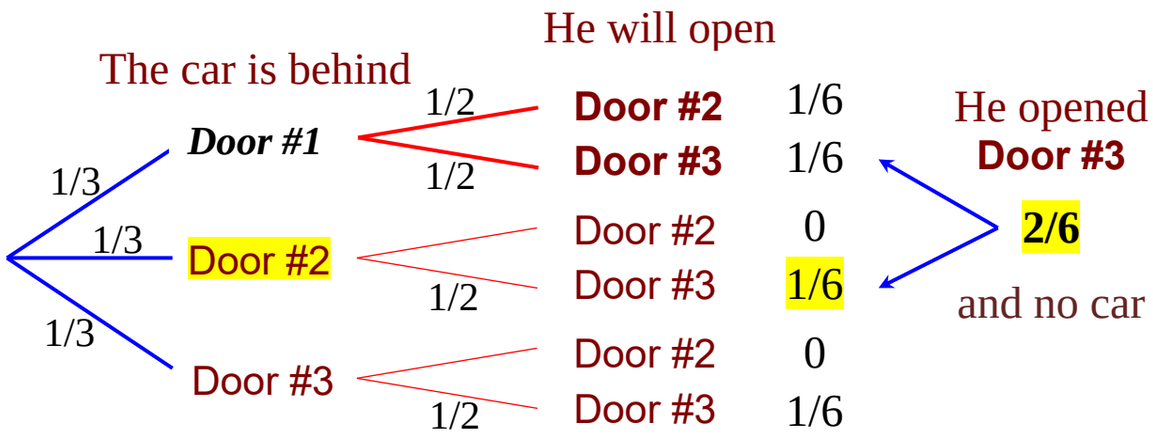
Suppose that everything is the same except that the game show host forgot to find out *in advance* which door has the car behind it.

In the spirit of "the show must go on," he makes a guess at which of the two doors to open and gets lucky, opening a door behind which stands a goat.

Now should the contestant switch?

* Variation: The game show host is drunk!

You chose Door #1		He will open		
		Door#1	Door#2	Door#3
If the car is behind	Door#1		1/2	1/2
	Door#2		1/2	1/2
	Door#3		1/2	1/2



- P[Car #1 | Opened #3] = 1/6 out of 2/6 = **1/2**
- P[Car #2 | Opened #3] = 1/6 out of 2/6 = **1/2** **it doesn't matter!**

Jailer's Dilemma

Ask Marilyn, *Parade Magazine*, (July 5, 1992), p. 23

Three prisoners on death row are told that one of them has been chosen at random for execution the next day, but the other two are to be freed.

One privately begs the warden to at least tell him the name of one other prisoner who will be freed. The warden relents: 'Chip will go free.'

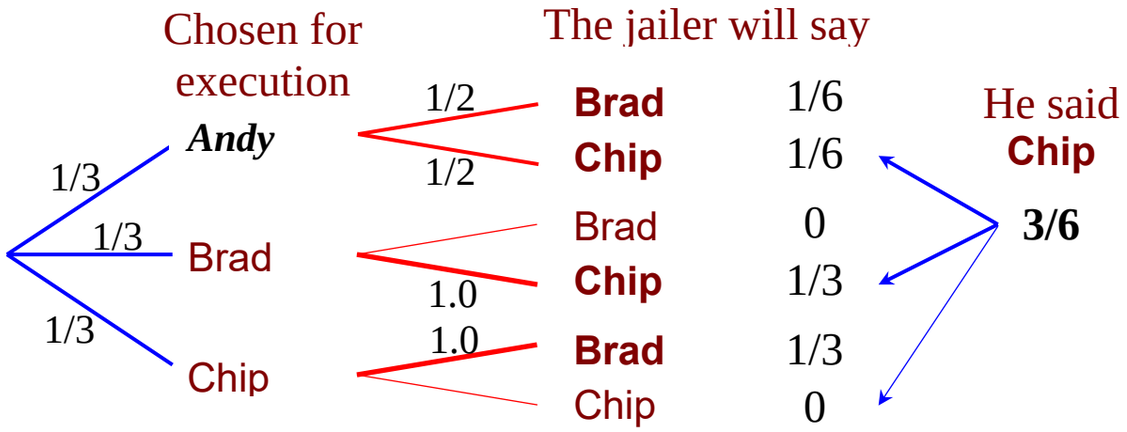
Horrified, the first prisoner says that because he is now one of only two remaining prisoners at risk, his chances of execution have risen from one-third to one half!

Should the warden have kept his mouth shut?

– Marvin M. Kilgo III, Camden, S. C."

* The Solution (No controversy at all !)

You're Andy		Who will be set free? The jailer will say		
		Andy	Brad	Chip
Chosen for execution	Andy		1/2	1/2
	Brad			1
	Chip		1	



- P[Andy will be executed | Chip will be free] = 1/6 out of 3/6 = **1/3**
- P[Brad will be executed | Chip will be free] = 1/3 out of 3/6 = **2/3**

The **rule** is clear!

Information Economics Approach

Young H. Chun, "On the Information Economics Approach to the Generalized Game Show Problem," *The American Statistician*, 53, (1999), pp. 43-51.

Ex] Railroad crossing

- Information structure

$$\mathbf{S} = \begin{matrix} \\ train \\ no\ train \end{matrix} \begin{matrix} \begin{matrix} red & green \end{matrix} \\ \begin{bmatrix} 0.9 & 0.1 \\ 0.4 & 0.6 \end{bmatrix} \end{matrix}$$

- Decision matrix

$$\mathbf{D} = \begin{matrix} \\ red \\ green \end{matrix} \begin{matrix} \begin{matrix} stop & go \end{matrix} \\ \begin{bmatrix} x & 1-x \\ 1-y & y \end{bmatrix} \end{matrix}$$

- Loss matrix

$$\mathbf{L} = \begin{matrix} \\ stop \\ go \end{matrix} \begin{matrix} \begin{matrix} train & no\ train \end{matrix} \\ \begin{bmatrix} 0 & 10 \\ 100 & 0 \end{bmatrix} \end{matrix}$$

- Prior probability matrix

$$\mathbf{P} = \begin{matrix} \\ train \\ no\ train \end{matrix} \begin{matrix} \begin{matrix} train & no\ train \end{matrix} \\ \begin{bmatrix} 0.2 & 0 \\ 0 & 0.8 \end{bmatrix} \end{matrix}$$

- Expected loss

Min z = trace (**SDLP**) = $-14.8x - 2.8y + 22.8$

Ex] Game Show Problem

- Information structure

to

$$\mathbf{S} = \begin{array}{c} \\ Car\ \#1 \\ Car\ \#2 \\ Car\ \#3 \end{array} \begin{array}{c} \begin{array}{ccc} Open\ \#1 & Open\ \#2 & Open\ \#3 \end{array} \\ \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \end{array}$$

- Decision matrix

$$\mathbf{D} = \begin{array}{c} \\ Open\ \#1 \\ Open\ \#2 \\ Open\ \#3 \end{array} \begin{array}{c} \begin{array}{ccc} Switch\ to\ \#1 & \#2 & \#3 \end{array} \\ \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix} \end{array}$$

- Loss matrix

$$\mathbf{L} = \begin{array}{c} \\ Switch\ to\ \#1 \\ \#2 \\ \#3 \end{array} \begin{array}{c} \begin{array}{ccc} Car\ \#1 & Car\ \#2 & Car\ \#3 \end{array} \\ \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{bmatrix} \end{array}$$

- Prior probability matrix

$$\mathbf{P} = \begin{array}{c} \\ Car\ \#1 \\ Car\ \#2 \\ Car\ \#3 \end{array} \begin{array}{c} \begin{array}{ccc} Car\ \#1 & Car\ \#2 & Car\ \#3 \end{array} \\ \begin{bmatrix} \pi_1 & & \\ & \pi_2 & \\ & & \pi_3 \end{bmatrix} \end{array}$$

- Expected loss

Min z = trace (**SDLP**)

* Microsoft Excel - Solver

Movie Trivia

It is based on the story about six MIT students
who were trained to become experts in card counting and
subsequently took Las Vegas casinos for millions in winnings.

21 (2008)

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