LSU E. J. OURSO COLLEGE OF BUSINESS

# Dr. Chun's Numb3rs & L∅gic

## *Breast Cancer?*

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# Chances are…

Steven Strogatz, *New York Times* (April 25, 2010)

*The probability that one of the women has breast cancer is 0.8%. If a woman has breast cancer, the probability is 90% that she will have a positive mammogram. If a woman does not have breast cancer, the probability is 7% that she will still have a positive mammogram. Imagine a woman who has a positive mammogram.*

*What is the probability that she actually has breast cancer?*

(a) More than 80%

(b) 50% ~ 80%

(c) 10% ~ 50%

(d) Less than 10%

(e) Absolutely no idea!

In one study, a German researcher and his colleagues asked doctors in Germany and the United States to estimate the probability that a woman with a positive mammogram actually has breast cancer, even though she's in a low-risk group: 40 to 50 years old, with no symptoms or family history of breast cancer.

To make the question specific, the doctors were told to assume the following statistics — couched in terms of percentages and probabilities — about the prevalence of breast cancer among women in this cohort, and also about the mammogram's sensitivity and rate of false positives:

*The probability that one of the women has breast cancer is 0.8%. If a woman has breast cancer, the probability is 90% that she will have a positive mammogram. If a woman does not have breast cancer, the probability is 7% that she will still have a positive mammogram. Imagine a woman who has a positive mammogram.*

*What is the probability that she actually has breast cancer?*

The German researcher describes the reaction of the first doctor he tested, a department chief at a university teaching hospital with more than 30 years of professional experience:

"He was visibly nervous while trying to figure out what he would tell the woman. After mulling the numbers over, he finally estimated the woman's probability of having breast cancer, given that she has a positive mammogram, to be 90%. Nervously, he added,

> *'Oh, what nonsense. I can't do this. You should test my daughter; she is studying medicine.'*

He knew that his estimate was wrong, but he did not know how to reason better. Despite the fact that he had spent 10 minutes wringing his mind for an answer, he could not figure out how to draw a sound inference from the probabilities."

When the German researcher asked 24 other German doctors the same question, their estimates whipsawed from 1% to 90%. Eight of them thought the chances were 10% or less, 8 more said 90%, and the remaining 8 guessed somewhere between 50 and 80%. (Imagine how upsetting it would be as a patient to hear such divergent opinions.)

As for the American doctors, 95 out of 100 estimated the woman's probability of having breast cancer to be somewhere around 75%.

**The right answer is 9.4%!**

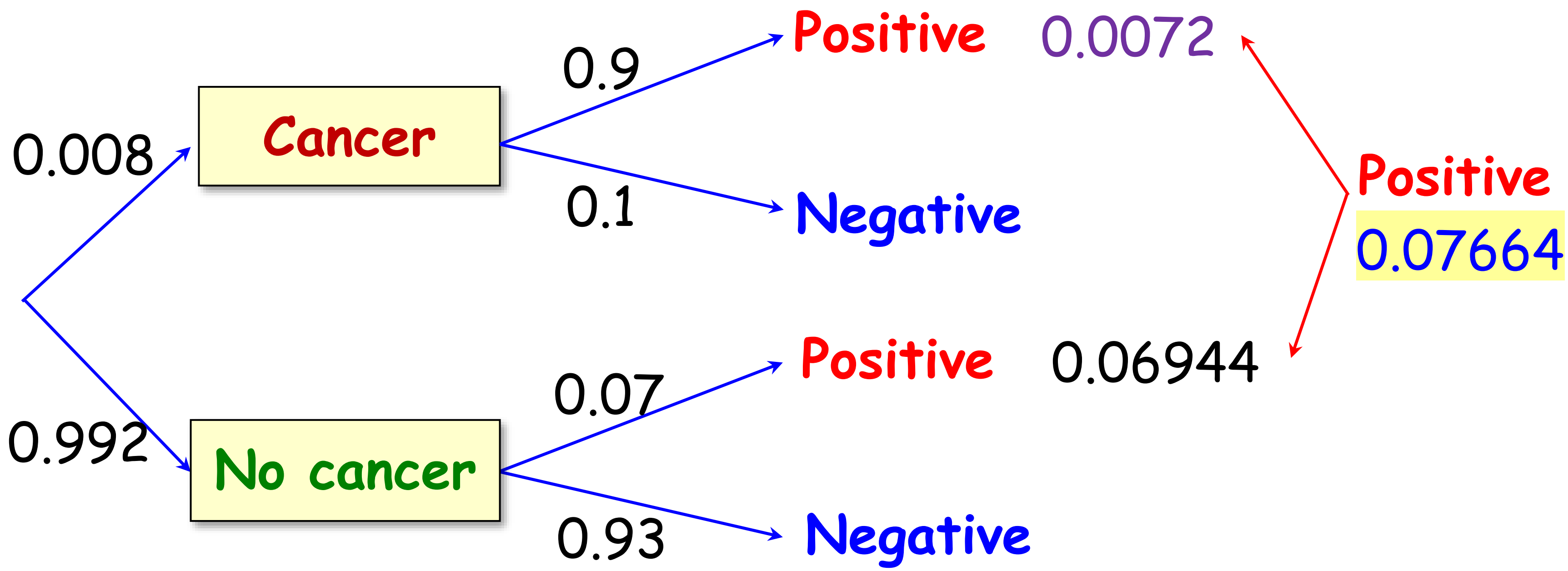
- **Contingency Table:**

| | Positive | Negative | Total |
|---|---|---|---|
| Cancer | 720 | 80 | 800 |
| No cancer | 6,944 | 92,256 | 99,200 |
| Total | 7,664 | 92,336 | 100,000 |

- P[**Cancer**?] = 800/100,000 = 0.8% (Prevalence rate!)
- P[Positive?|**Cancer**!] = 720/800 = 90% (Effective rate!)
- P[Positive?|**No cancer**!] = 6,944/99,200 = 7% (False-positive!)

- P[**Cancer**?| Positive!] = P[ **Cancer** & Positive] / P[Positive]
  = 720 / 7,664
  = **9.4%**

- Tree diagram:




- P[Cancer?| Positive!] = P[ Cancer & Positive] / P[Positive]

  = 0.0072 / 0.07664

  = **9.4%**

- **Multiple testing?**

Take the same blood test multiple times!

| Number of negative results, *y* | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **8** | 0.000% | 0.000% | 0.000% | 0.000% | 0.000% | 0.005% | 0.065% | 0.830% |
| **7** | 0.000% | 0.000% | 0.000% | 0.000% | 0.004% | 0.047% | 0.602% | 7.222% |
| **6** | 0.000% | 0.000% | 0.000% | 0.003% | 0.034% | 0.436% | 5.330% | 41.993% |
| **5** | 0.000% | 0.000% | 0.002% | 0.025% | 0.316% | 3.913% | 34.368% | 87.068% |
| **4** | 0.000% | 0.001% | 0.018% | 0.229% | 2.862% | 27.471% | 82.964% | 98.428% |
| **3** | 0.001% | 0.013% | 0.165% | 2.086% | 21.505% | 77.888% | 97.840% | 99.829% |
| **2** | 0.009% | 0.120% | 1.518% | 16.540% | 71.815% | 97.038% | 99.763% | 99.982% |
| **1** | 0.087% | 1.103% | 12.537% | 64.826% | 95.951% | 99.673% | 99.974% | 99.998% |
| **0** | 0.800% | 9.395% | 57.139% | 94.487% | 99.548% | 99.965% | 99.997% | 100.000% |
| | **0** | **1** | **2** | **3** | **4** | **5** | **6** | **7** |

Number of positive results, *x*

Stop and Conclude No Disease!

Stop and Conclude Disease!

- **Decision rule**: Continue if 0.500% < Probability < 80.000%

# Risk of false alarm from mammogram is 50% Over Decade

*New York Times* (April 16, 1998)

> A recent study concluded that women who receive mammograms every year for a decade run a 50-50 chance of receiving a false-positive result.

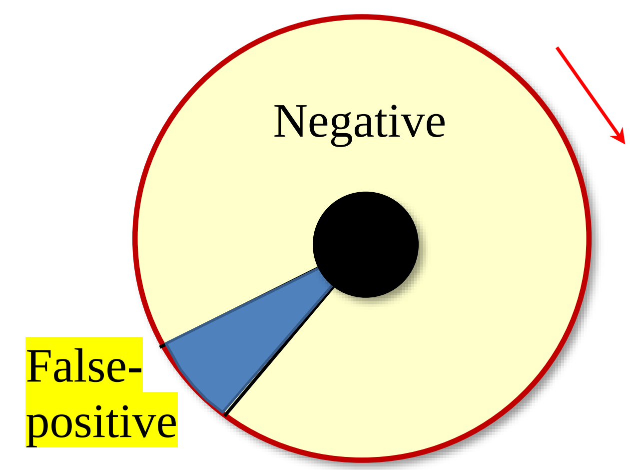
- Suppose that P[False-positive] = P[Positive| No cancer] = 0.07
- Binomial random variable with $n$=10 and $p$=0.07

  P[ $X$ number of false-positives | $n$=10, $p$=0.07]

  $= C(10, x) * 0.07^{x} * 0.93^{(10-x)}$

- At least one false-positive?

  $P[X \geq 1] = 1 - P[X = 0] = 1 - 0.93^{10} =$ 51.6%




Spin the wheel and throw your dart!

## * Are you suffering from innumeracy?

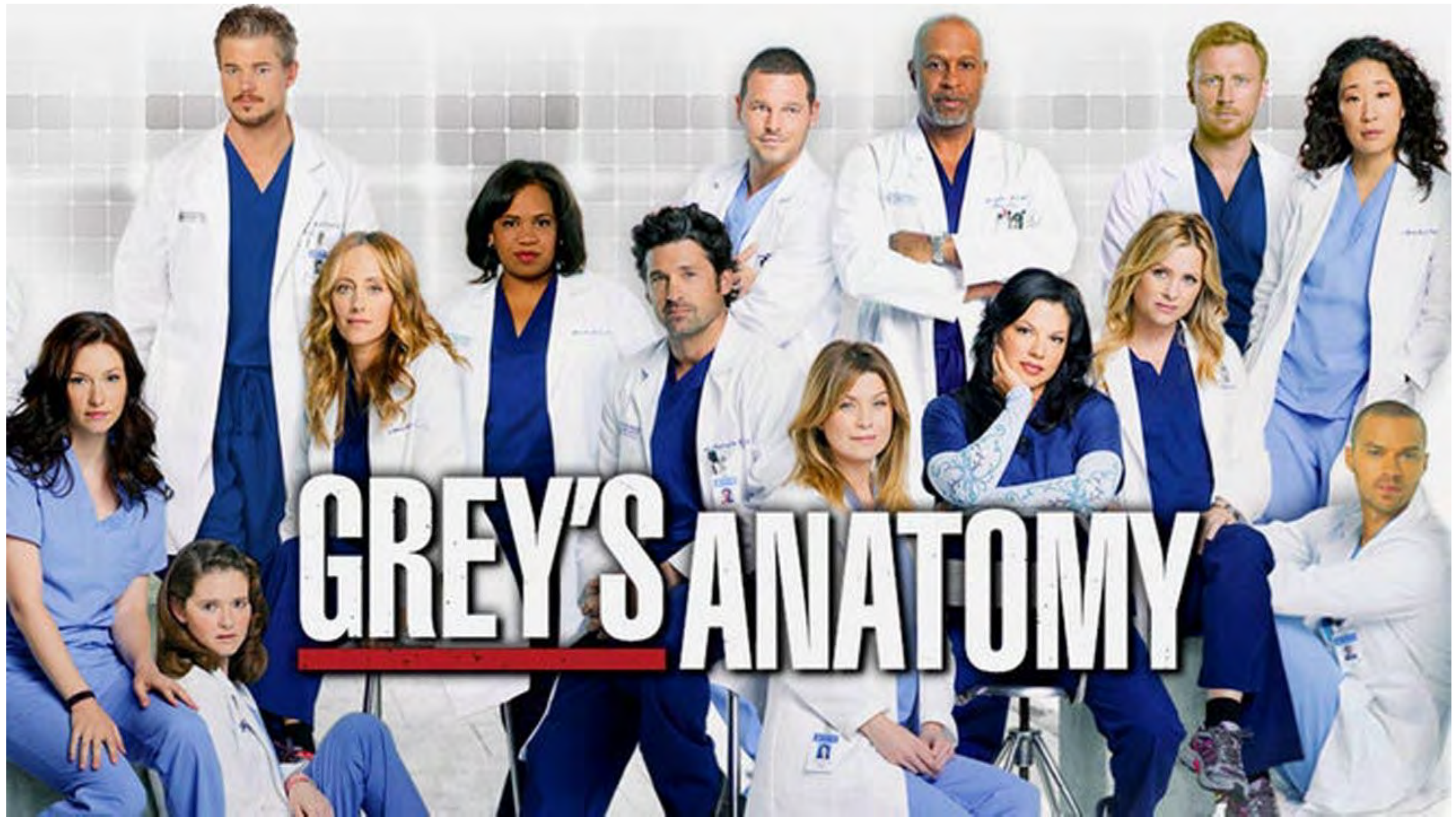
- Probability — P[Cancer?] P[Win?]
- Conditional probability — P[Cancer? | Positive! ] P[Win? | Home game!]
- Random variable — Number of false-positive results; Number of wins this season
- Probability distributions — Binomial distribution; Normal distribution, etc.

Innumeracy? An inability to deal comfortably with the fundamental notions of number and chance, plagues far too many otherwise knowledgeable citizens.

# TV Drama Quiz?


The medical drama series focuses on the personal and professional lives of five surgical interns and their supervisors at a hospital in Seattle.

# TV Drama Quiz!




The medical drama series focuses on the personal and professional lives of five surgical interns and their supervisors at a hospital in Seattle.