LSU E. J. OURSO COLLEGE OF BUSINESS

Dr. Chun's Numb3rs & L∅gic

Dorfman Group Testing

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From Leo Rosten. *Giant Book of Laughter*, New York, 1985, P. 463.

Years ago, old Mr. Lyle carried a milk bottle, filled to the brim with yellow fluid, to the Ballantrae Polyclinic and Health Centre. He went into the clinic and handed the bottle carefully to a nurse,

"Here it is, Nurse Cameron. The urine specimen. Name of Fergus Lyle…"

"Aye. Coom back in three days for the analysis."

In three days, Mr. Lyle returned. He was taken to see Dr. Howard, a young doctor. The doctor glanced down a report and said,

"Well, Mr. Lyle, the laboratory report they gie ye is all fine. Complete bill o' health. No diseases."

Mr. Lyle went home with great speed and scarcely contained delight. Before he was through the door of his house, he cried,

"Kathy, lass, Kathy! It's great news I am bringin' ! "

His wife hurried in from the kitchen, "Aye?"

"Hoot, Kathy, it's fine I am in all respects! And so are ye, and so is y'r old dad, an' y'r brother Nairn, and bonnie cousin Molly, and also me brother Clyde, an' his woman, an' my old friend Douglas, an' th' charwoman…"

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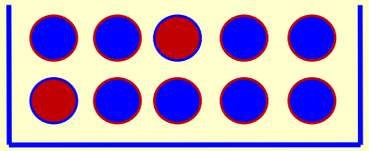
* Problem

During the World War II, the wartime draft used Wasserman screening to weed out all syphilitic men among those called up for induction into the US Armed Forces. Is there any efficient method for screening those syphilitic men from a certain large population, rather than doing millions of tests?

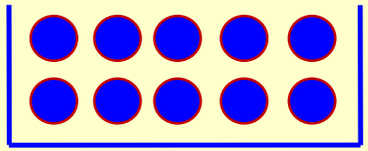
* Solution: Dorfman group testing

Instead of testing each blood sample individually, Robert Dorfman (1943) proposed to pool n samples in a single analysis. Presence of syphilitic antigen in the pool leads to a decision to make n individual tests; absence of such an antigen leads to immediate clearance of all n men without further testing.

- **Stage 1:**
  Test the group of *n* individual samples



If tested positive



If tested negative

- **Stage 2:**
  Each sample is further tested individually.

No more tests are needed.

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* Efficiency of the Group Testing

n = group size

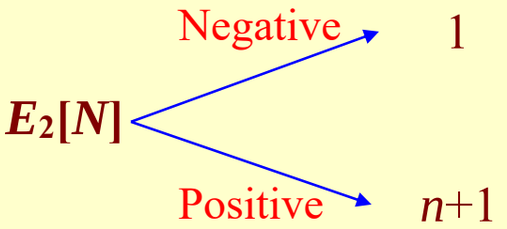
p = prevalence rate of the disease

E[N] = average number of tests

- **Case 1: Individual Testing**

$E_1[N] \longrightarrow n$

- **Case 2: Group Testing**



- P[Negative] = $(1-p)^n$
- P[Positive] = $1-(1-p)^n$
- $E_2[N] = (1-p)^n * 1 + [1-(1-p)^n] * (n+1) = n[1-(1-p)^n] + 1$
- **Ratio** = $E_2[N] / E_1[N] = [1-(1-p)^n] + 1/n$
- If p is very small, then this ratio becomes very small, which implies that the group testing is more effective!

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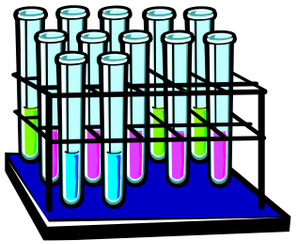
* Example



Prevalence rate $p = 0.01$

Group size $n = 10$

Cost per testing $1

- **Case 1: Individual testing**

  $E_1[N] = \$10$

- **Case 2: Group testing** with $n=10$

  $P[\text{Negative}] = (1-p)^n = 0.904$

  $P[\text{Positive}] = 1-(1-p)^n = 0.096$

  $E_2[N] = n\,[\,1-(1-p)^n\,] + 1 = \$1.96$

- Ratio = $E_2[N] / E_1[N]$

  $= \$1.96/\$10 = 19.6\%$

Why $n=10$? Can I perform a **group test** with $n=20$, or with $n=5$?

How to determine the optimal group size?

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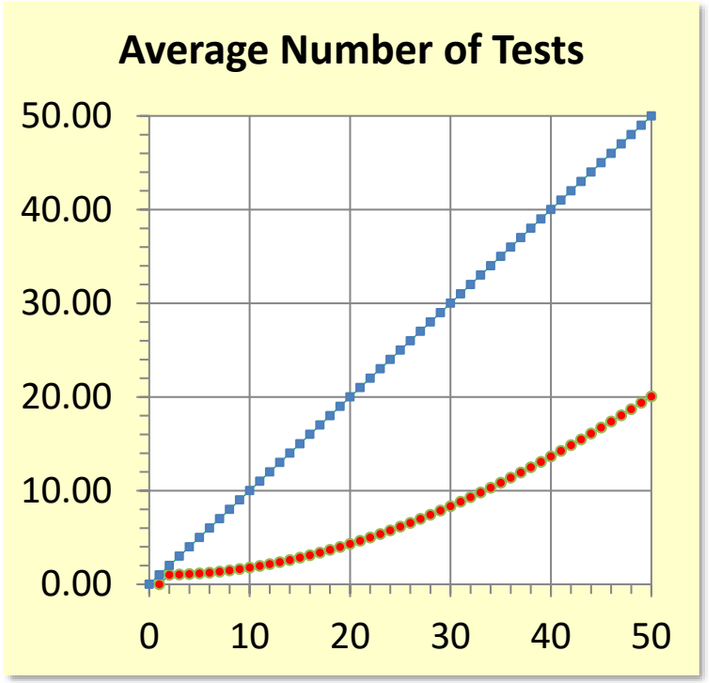
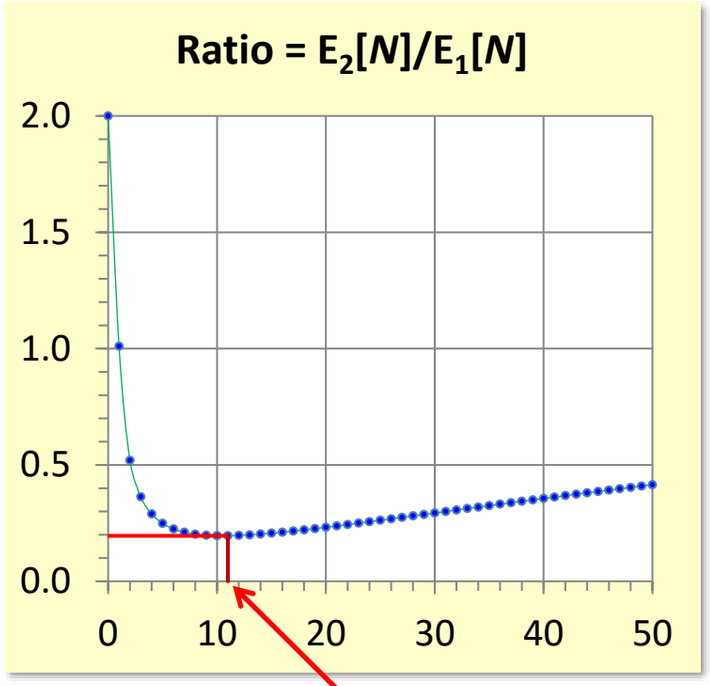
* Optimal Group Size n^*

For a given p, find n that minimizes the ratio,

$$\text{Ratio} = E_2[N] / E_1[N] = [1-(1-p)^n] + 1/n$$

Ex] Suppose p = 0.01





The minimum ratio is 19.557% at the optimal group size, $n^* = 11$

Excel?

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Movie Trivia

It follows the story of two best friends, Rafe and Danny, and their love lives as they go off to join the Pacific war.

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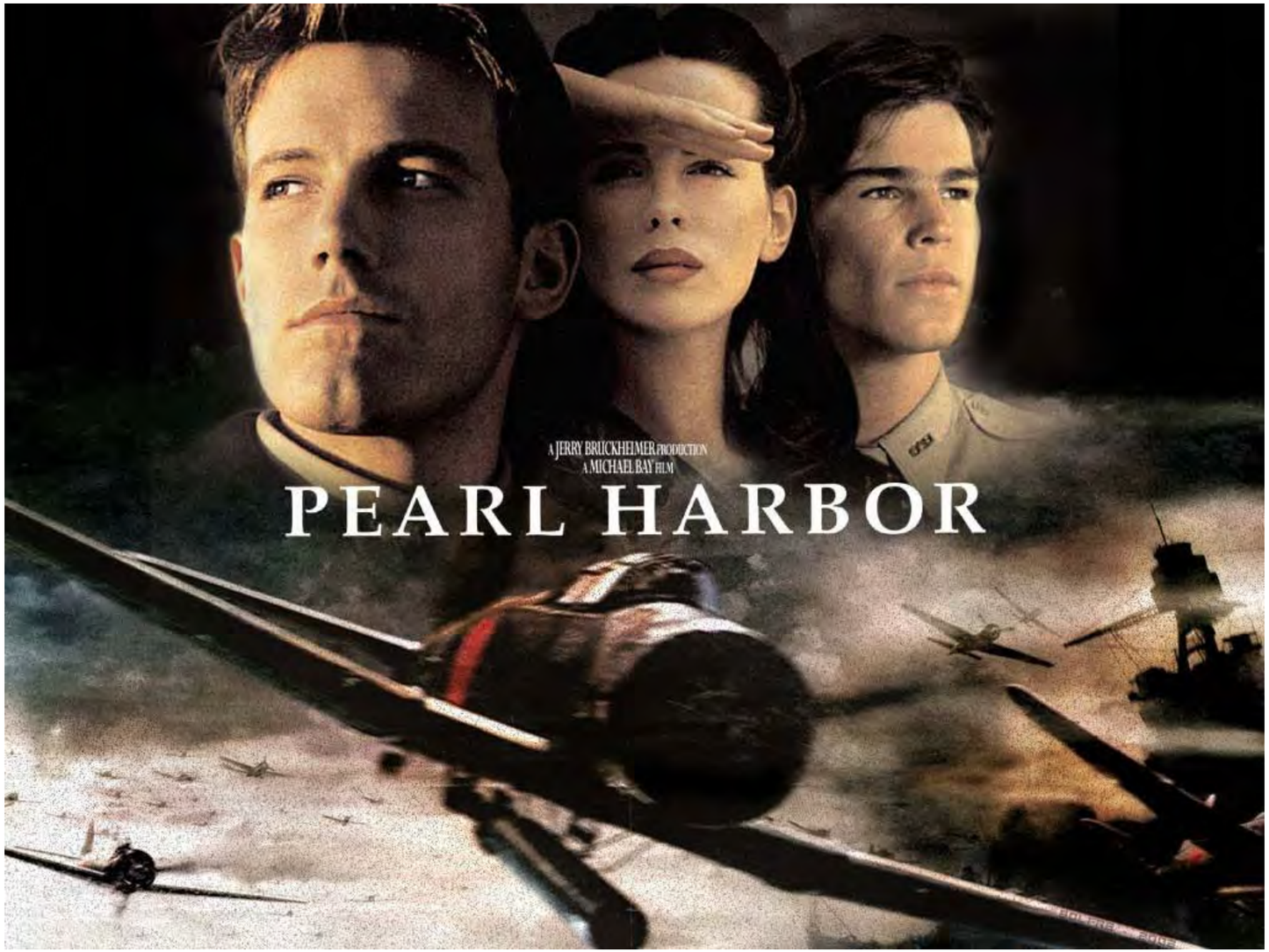
Pearl Harbor (2001)



It follows the story of two best friends, Rafe and Danny, and their love lives as they go off to join the Pacific war.

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