

Dr. Chun's Numb3rs & Løgic

Number of Questions



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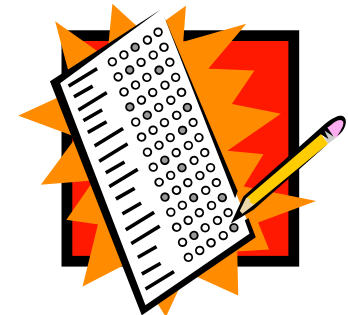
Number of Questions

Ask Marilyn, Parade Magazine (March 25, 2007), Page 21.



"I am an adjunct professor at a college, and I sometimes give students the **option** of taking a **10-question exam** (each question = 10 points) or a **20 question exam** (each question = 5 points). They almost always choose the **latter**, believing they have a better chance of scoring higher.

I don't teach **statistics**: Is this logic sound?
(Note: The questions are **multiple-choice**.)"



Ellen Bourhis Nolan, Cornwall on Hudson, N. Y.



■ Notation

n = Total number of questions in the exam,

m = Number of correct answers among n ,

X = Score in the exam ($= 100/n * m$), and

p = Probability that your answer is correct.



■ Random Variable

(1) m is a binomial random variable with n and p .

$$E[m] = np \quad \text{and} \quad \text{Var}[m] = np(1-p)$$

(2) X is a random variable ($= 100/n * m$) with

$$\begin{aligned} E[X] &= E[100/n * m] \\ &= E[m] * 100/n = np * 100/n = 100p \end{aligned}$$

$$\begin{aligned} \text{Var}[X] &= \text{Var}[100/n * m] \\ &= \text{Var}[m] * (100/n)^2 = np(1-p) * (100/n)^2 = (100)^2 p(1-p)/n \end{aligned}$$

■ $E[X]$ and $\text{Var}[X]$ depend on p for a given n !



- Notation

n = Total number of questions in the exam,

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p = Probability that your answer is correct.

- Expected Value and Variance of X

p	$n = 1$ question		$n = 10$ questions		$n = 20$ questions	
	$E[X]$	$Var[X]$	$E[X]$	$Var[X]$	$E[X]$	$Var[X]$
0.0	0	0.0	0	0.0	0	0.0
0.1	10	900.0	10	90.1	10	45.0
0.2	20	1600.0	20	160.0	20	79.9
0.3	30	2100.4	30	210.0	30	105.1
0.4	40	2400.0	40	239.9	40	119.9
0.5	50	2500.0	50	250.0	50	125.0
0.6	60	2400.0	60	239.9	60	119.9
0.7	70	2100.4	70	210.0	70	105.1
0.8	80	1600.0	80	160.0	80	79.9
0.9	90	900.0	90	90.1	90	45.0
1.0	100	0.0	100	0.0	100	0.0



■ Notation

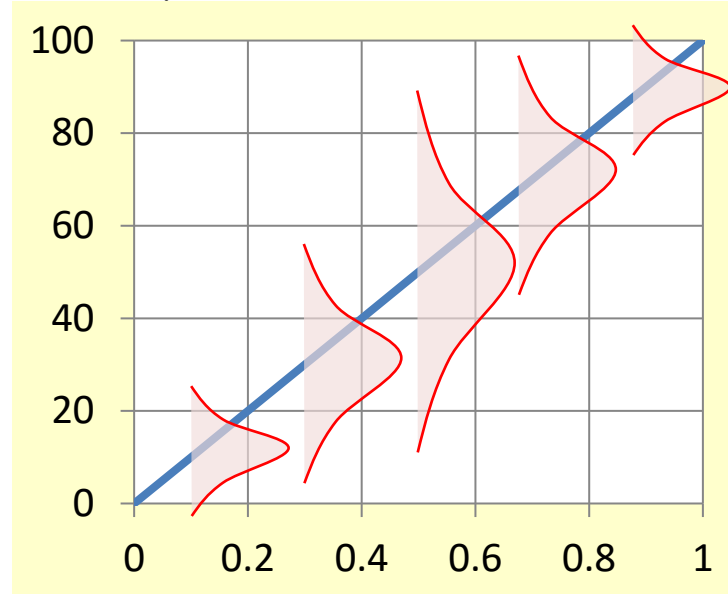
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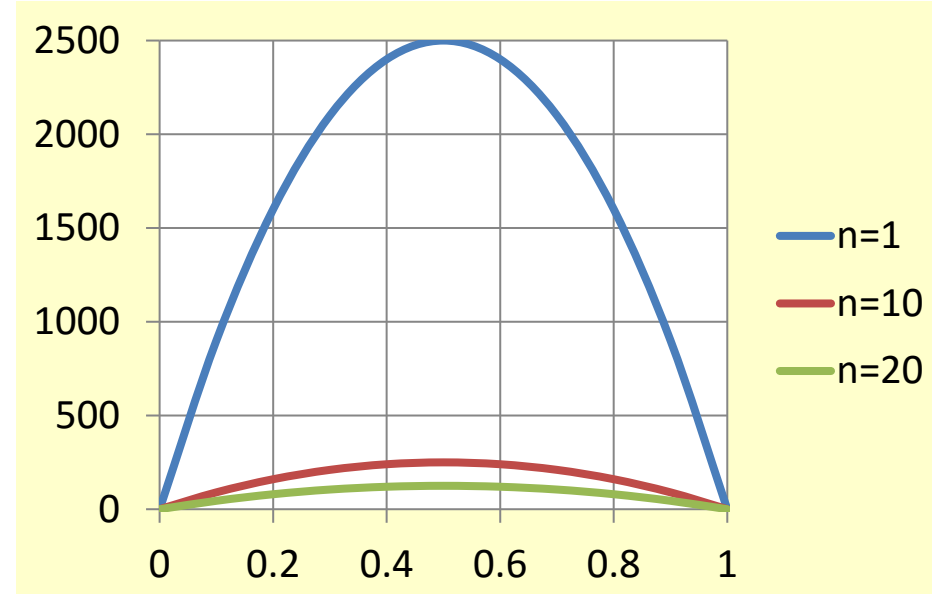
p = Probability that your answer is correct.

Expected Value, $E[X]$



p

Variance, $\text{Var}[X]$



p

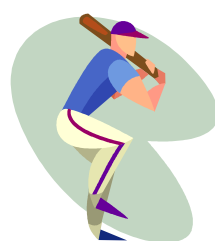
- In the ideal situation, what you deserve (p) is what you get $E[X]$ with no variation!

The Scoring Problem

Richard Bookstaber, *The Wall Street Journal*, (July 10, 2010)



There is an **analytical basis** for determining the amount of scoring a sport should have. The greater the number of scores in a sport, the lower the chance for a **lucky win** by a team that is **inferior**.



Thus, a sport should have enough scoring — but no more than enough scoring — so that (a) a team that, in a large sample of games, tends to **lose** to most everyone usually doesn't beat a team that tends to **beat** everyone, and (b) any one **player error** or **referee call** typically will not change the **outcome**.

On this basis, it seems, **soccer** and **hockey** have too **few** scores, **basketball** and **tennis** have too **many**, and **baseball** and **American football** are somewhere near the **sweet spot**.

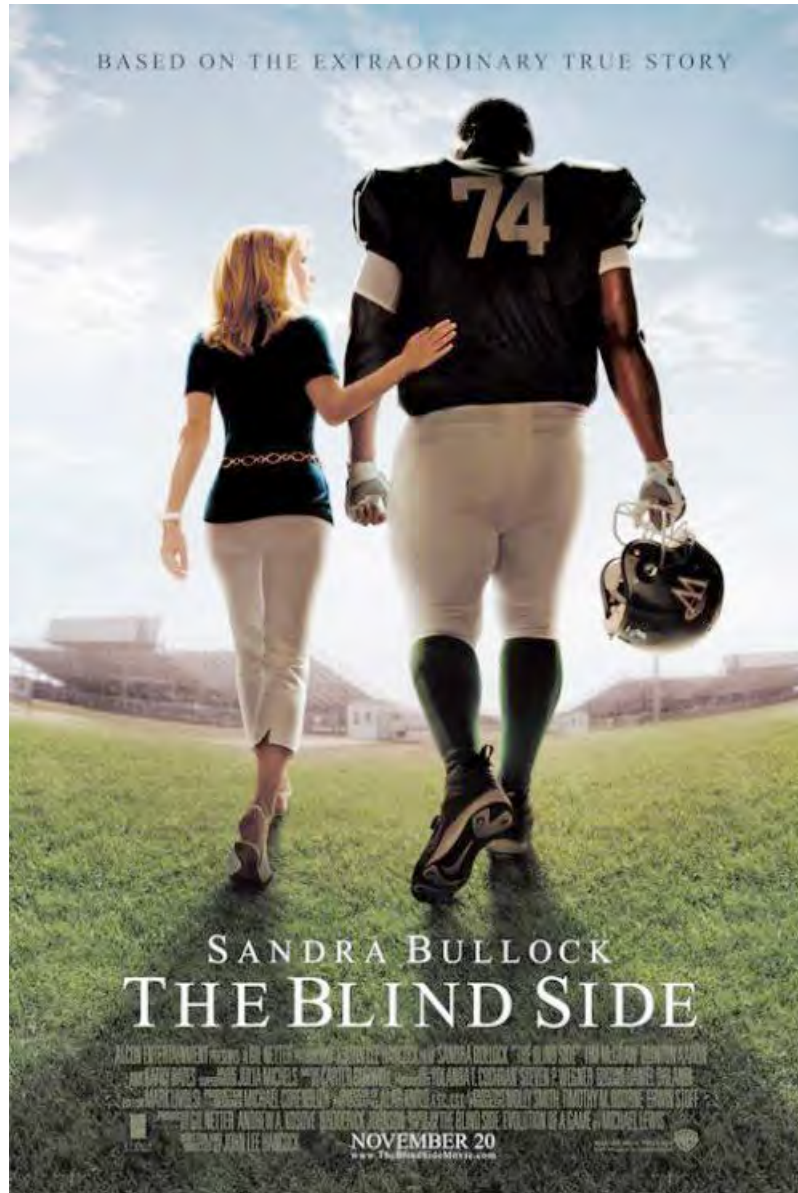


Movie Trivia



The story of Michael Oher, a homeless and traumatized boy who became an All American football player and first round NFL draft pick with the help of a caring woman and her family.

The Blind Side (2009)



The story of Michael Oher, a homeless and traumatized boy who became an All American football player and first round NFL draft pick with the help of a caring woman and her family.