

Dr. Chun's Numb3rs & Løgic

What's 2/3 of the Average?



Young H. Chun, Ph.D.

*Professor of Decision Science &
Cherie H. Flores Endowed Chair of MBA Studies*



What's $2/3$ of the Average?

Suppose everyone in the class selects a **real number** between **0** and **100**, inclusive (i.e. **0** and **100** are both possible choices, as is any other number between).

The **winner** is the individual (or individuals) who selects the number **closest** to $2/3$ of the **average of numbers** chosen.

What number do you choose? Why?

Suppose there are three students who select the numbers **20**, **40**, and **60**. The **average** is **40**, and $2/3$ of the **average** is **26.66**.

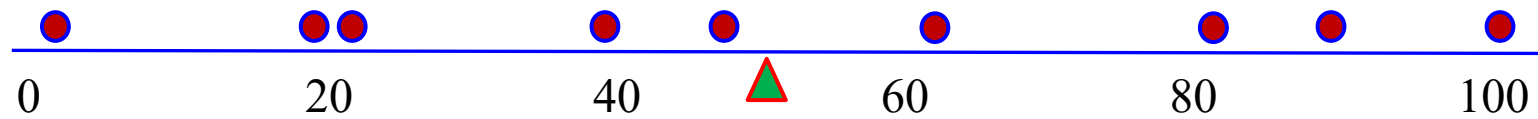
Therefore, the winner is the student who selected **20** because it is **closest** to $2/3$ of the **average**.

Ex] Excel experiment

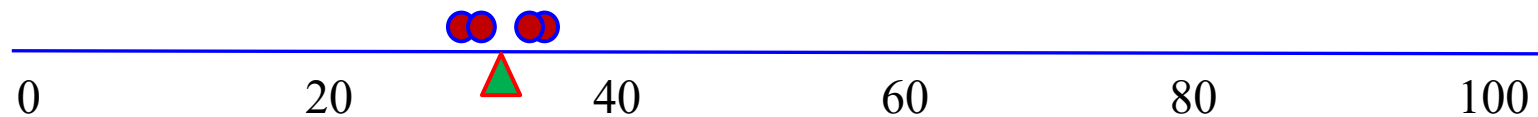




- The lowest, 'Level 0' players, choose numbers randomly from the interval $[0, 100]$.

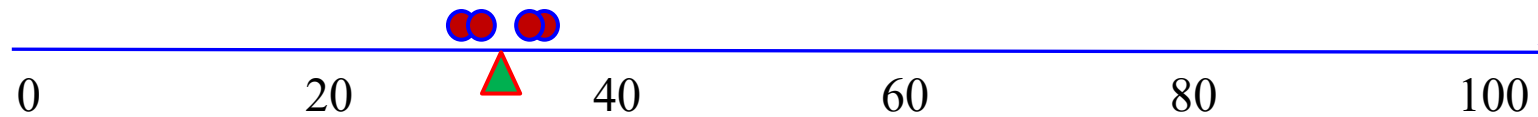


- The next higher, 'Level 1' players believe that all other players are Level 0 and the average of all numbers submitted should be around 50. Thus, these Level 1 players choose, as their number, 2/3 of 50, or 33.

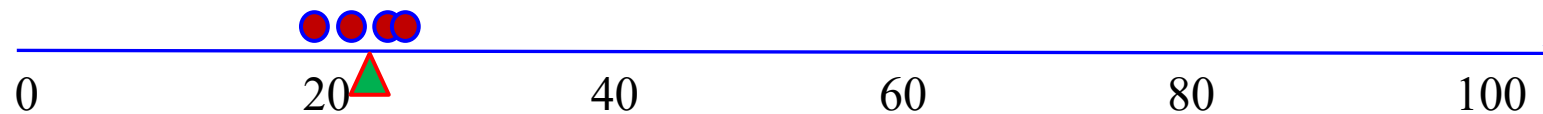




- 'Level 1' players choose, as their number, 33.



- Similarly, the next higher 'Level 2' players believe that all other players are Level 1 players. These Level 2 players therefore reason that the average of all numbers submitted should be around 33, and so they choose, as their number, $2/3$ of 33 or 22.

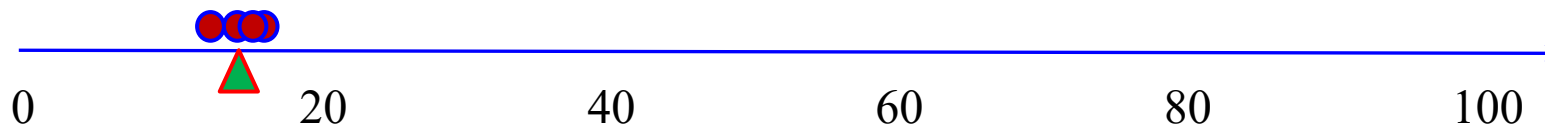




- 'Level 2' players choose, as their number, $2/3$ of 33 or 22.



- The next higher 'Level 3' players play a best response to the play of Level 2 players and so on.





The process of recursively eliminating sets of numbers that would be **irrational** to select is known as **iterative elimination of weakly dominated strategies**. The case that everyone chooses **zero** is the **Nash equilibrium**, which means nobody would **regret** their choice.

Do you think the **winning choice** would really be **zero**? I'll bet not. Why?

Because it is unlikely that everyone in the class would follow the **chain of logic** described above: Everyone may be **rational**. Everyone may assume everyone is **rational**. Everyone may assume that everyone assumes that everyone is **rational**.

But at some point, some people implicitly are going to stop the sequence. In fact, on average, people only nest these assumptions to **three** levels. Evidence: **21.6** was the **winning number** in such a game with **19,196** participants organized by a Danish newspaper.

This example illustrates the **difference** between **perfect rationality** and **common knowledge**.

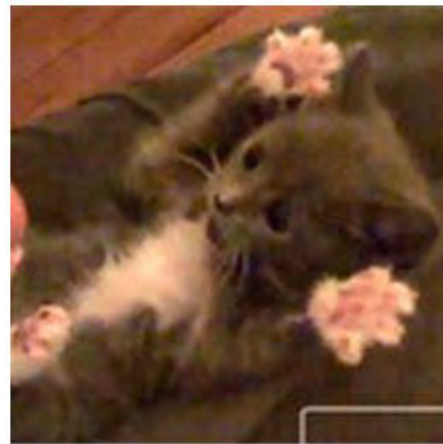


Ranking Cute Animals: A Stock Market Experiment

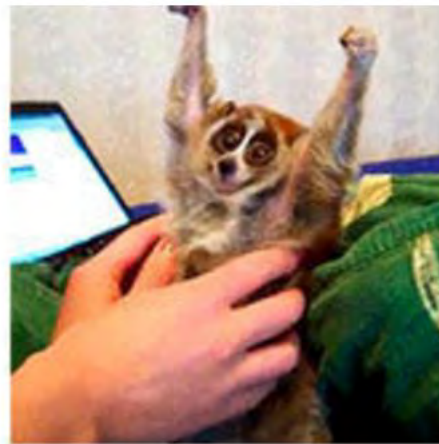
David Kestenbaum, *Planet Money*, NPR (January 14, 2011)



National Public Radio's *Planet Money* tested the theory by having its listeners select the cutest of **three animal videos**. The listeners were broken into **two** groups. One selected the animal **they thought was the cutest**, and the other selected the one **they thought most participants would think was the cutest**.



Kitten



Slow Loris



Baby Polar Bear





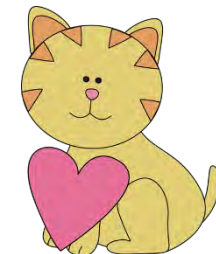
The results showed significant **differences** between the two groups. **50%** of the first group selected a video with a **kitten**, compared to **76%** of the second group selecting the same video.

Individuals in the second group were generally able to disregard their **own preferences** and accurately make a decision based on the **expected preferences of others**.

$H_0: \pi_a = \pi_b$ $H_1: \pi_a \neq \pi_b$	Kitten	Slow loris	Baby polar bear
▪ Group A: They thought was the cutest	50%	27%	23%
▪ Group B: They thought most participants would think was the cutest.	76%	15%	9%

Marla Wood, a **Planet Money** listener from Colorado, was assigned to the second group. She thought the **loris** was the cutest, but she picked the **cat**, because she thought that's what **everybody else** would pick.

The results were considered to be consistent with **Keynes' theory**.



Keynesian Beauty Contest



Back in 1936, John Maynard Keynes described the action of **rational agents** in a stock market using an analogy based on a fictional **newspaper contest**, in which entrants are asked to choose the six most **attractive faces** from a hundred photographs. Those who picked **the most popular faces** are then eligible for a prize. Thus, to maximize the chances of winning the prize, you're not supposed to say **who you think is prettiest**. You're supposed to guess **who everyone else will think is the prettiest**.

In the market, Keynes argued, it doesn't make sense to invest in the **company you think is the best**. It makes sense to invest in the **company** that **you think other people will think is the best**. Because if everyone else invests in a company, the price of its stock will rise.

Of course, when everyone does this, it leads to a **slippery investment world**. "We have reached the **third degree** where we devote our intelligences to anticipating what average opinion expects the average opinion to be," Keynes wrote.



Movie Trivia



An FBI agent (Sandra Bullock) must go undercover in the Miss United States **beauty pageant** to prevent a group from bombing the event.

Miss Congeniality (2000)



SANDRA BULLOCK
MICHAEL CAINE
BENJAMIN BRATT
AND
CANDICE BERGEN

Miss
CONGENIALITY

"Sandra Bullock
Scores a Comic
Bull's-Eye!"
- David Sheehan, CBS-TV



An FBI agent (Sandra Bullock) must go undercover in the Miss United States **beauty pageant** to prevent a group from bombing the event.