

Dr. Chun's Numb3rs & Løgic

Sequential Decision Problems



Young H. Chun, Ph.D.

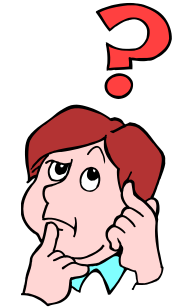
*Professor of Decision Science &
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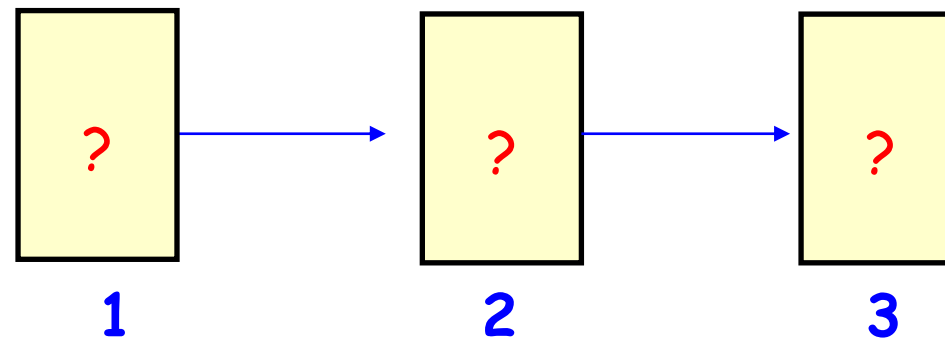
Three Slips of Paper

Tom and Ray Magliozzi, *Car Talk*, (Feb. 19, 2001)

Numbers are written on **three slips of paper**, which are then shuffled and placed face down on the table. You are allowed to **pick up** the pieces of paper one at a time and **stop** when you think you have the **largest number**.



Once you have rejected a number you cannot go back to it. Is there a **strategy** that gives you better than the **1/3** chance you have when you just choose a piece of paper **at random** and say this is the biggest?





Secretary Problem

Suppose that you are an employer who must hire a new person to fill an available position from n candidates for the position. The candidates will appear in a random order to be interviewed by you, and you will interview them **sequentially**, i.e., one at a time, in order in which they appear.

After interviewing each candidate, you must decide immediately whether or not you want to hire that candidate. If you decide to **hire** that candidate, the process terminates. If you decide **not to hire** that candidate, then you proceed to interview the next one. The candidate you have just interviewed leaves and, we assume, accepts some other position.



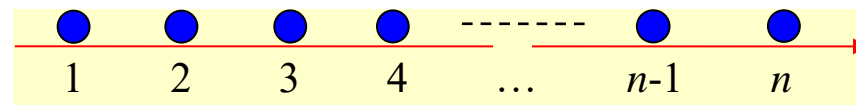
Thus, once you have decided not to hire a candidate, you cannot later change your mind. How could you select the **best candidate**?

- **Young H. Chun**, "Selecting the Best Choice in the Weighted **Secretary Problem**," *European Journal of Operational Research*, Vol. 92 (1996), pp. 135-147.
- **Young H. Chun**, "On the Two Extensions of the **Secretary Problem**," *Proceedings in the Northeast Decision Sciences Institute Annual Meeting*, Boston, April 1-3, 1992, 225-227



* Secretary problem

Optimal selection strategy
with *sequentially* presented alternatives over time.



Ex] Hiring a "secretary" - or a faculty member

1. *One* position is available.
2. Number of applicants is *known*.
3. Interviewed *sequentially* in random order, each order being *equally likely*.
4. Once rejected, *never recalled*.
5. Can *rank* from best to worst without tie.

* Bachelor's Dilemma

- Marry too early => Best choice has not yet come.
- Marry too late => Best choice has "*gone with the wind*"!



* Also known as...

- Best choice problem
- Bachelor's dilemma
- Beauty contest problem
- Dowry problem
- Candidate problem
- **Parking spot** problem: Closer to the entrance.
- Apartment problem: Find an apartment within a time limit.
- Tourist problem: Find the best motel on the highway.
- **House selling** problem: Find the highest bid.
- Job search problem
- **Streetwalker**'s dilemma: Find the highest offer?
- Marriage problem: When to stop searching and propose marriage.



* **Popular** - but not in the **decision science area**

"Constitute a '**field**' of study within mathematics, probability, and optimization."



* Classification

1. Based on the availability of information

- Let X_j be the value of the j th choice, $j=1,2,\dots,n$.
- X_j is an *i.i.d.* random variable from $F(\theta)$.



(a) Full information problem

- $F(\theta)$ is completely known *a priori*.

Ex] House selling problem: $F(\theta)$ is *Normal*(\$200k, \$10²k)

Ex] *Hue Hefner's* problem: You know everything about women!

(b) No information problem

- $F(\theta)$ is completely unknown. Rank-based selection.

Ex] Secretary problem: Qualifications are completely unknown.

Ex] "*The 40 Year Old Virgin*": He knows nothing about women.

(c) Partial information problem

- F is known, but θ is unknown. Bayesian selection.

Ex] You are learning quickly after each date.

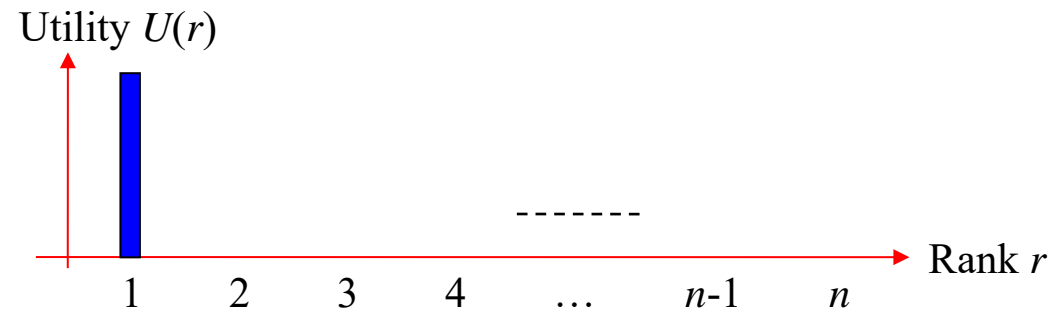


2. Based on the decision-maker's objective

(a) **Best choice** problem (all-or-nothing):

$$U(r) = \begin{cases} 1 & \text{if } r = 1 \\ 0 & \text{otherwise} \end{cases}$$

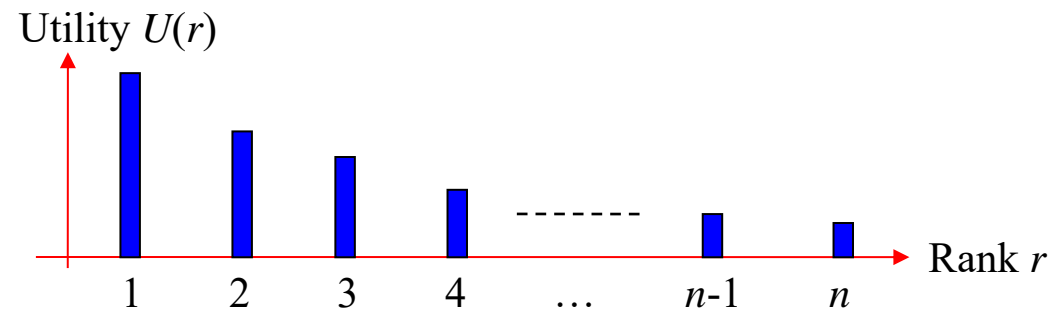
where r is the absolute rank of your **selected choice**.



- Maximize the **probability** of selecting the **best**.

(b) **Minimum rank** problem:

$$U(r) = \frac{1}{r}, \quad r = 1, 2, 3, \dots, n$$

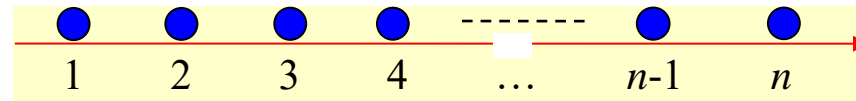


- Minimize the **expected rank** of the **selected choice**.



3. Based on the arrivals of choices

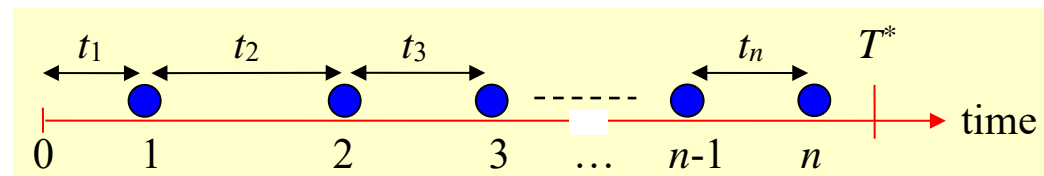
(a) Discrete-time case



- n alternatives are presented sequentially.
- n is known *a priori*.

Ex] Secretary problem, Parking stop problem

(b) Continuous-time case:



- Select a choice within a limited time T^* .
- Inter-arrival time t_j is a *random variable* from $F(\cdot)$, where F may be known *a priori* or can be estimated.

Ex] Moving sale, House selling problem

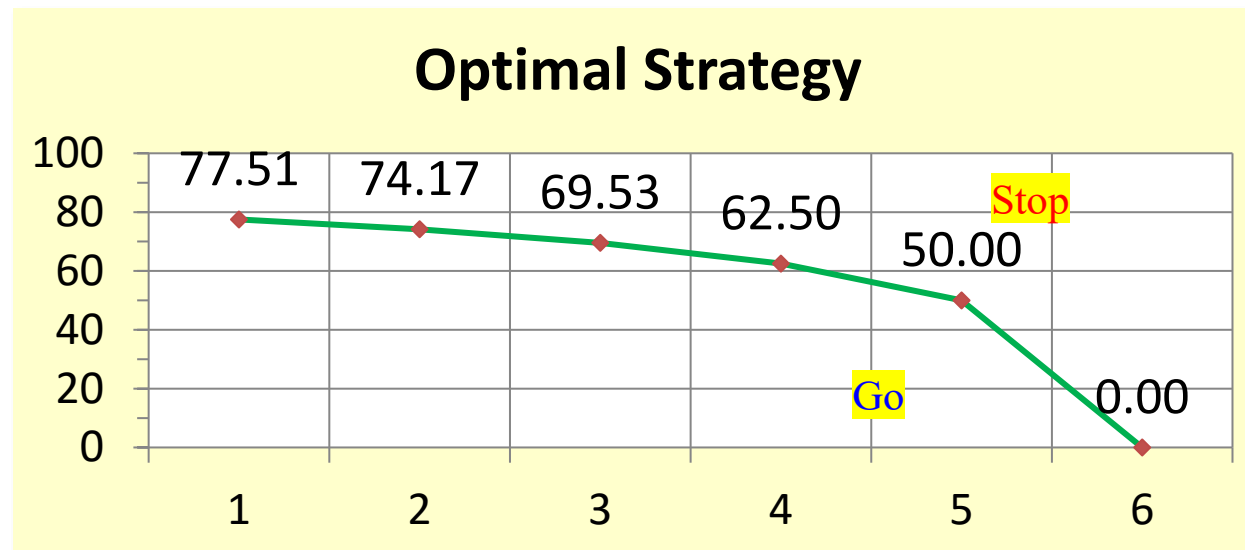


Full-Information Maximum Value problem

* Selection strategy

- Draw your **aspiration level** v_i , which is **decreasing** in time
- Select the **first choice** whose value is higher than the **level**.
- **House selling problem** with $F(x) \sim \text{Uniform}(0, 100)$

- Aspiration level: $v_i = \frac{1}{2}(1 + v_{i+1}^2)$



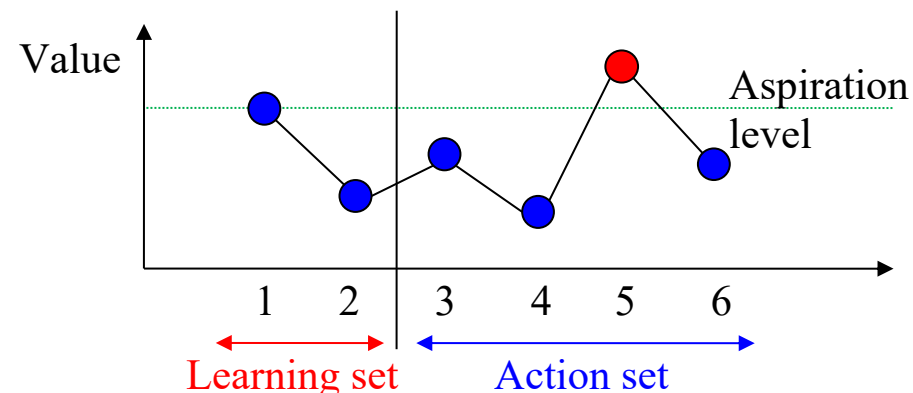
Excel #1

No-Information Best Choice problem



* Selection strategy

- Divide the entire set into the *learning* set (L) and the *action* set (A).
- Just observe the values of choices in the *learning set* without choosing any choices. The *highest value* in the *learning set* is your *aspiration level*.
- Select the *first choice* in the *action set* A whose value is higher than the *aspiration level*.

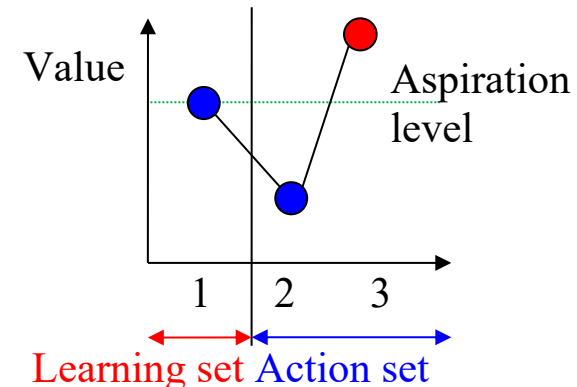
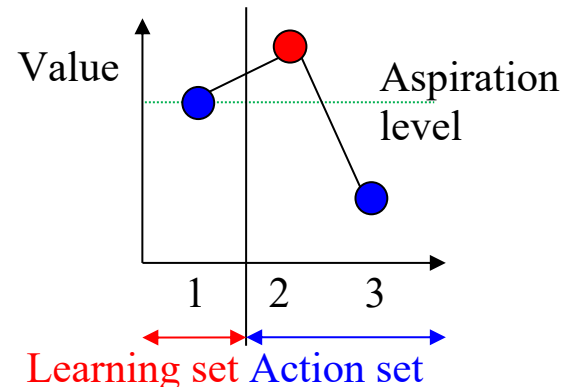


- When $n = 6$, $L = \{1, 2\}$ and $A = \{3, 4, \dots, 6\}$.
- The probability of selecting the *best choice* is $\pi^*(6) = 42.78\%$.



Ex] Three Slips of Paper

- When $n=3$, it can be shown that $L = \{1\}$ and $A = \{2, 3\}$.
- "Never take the **first** offer!"



- For $n = 3$, the **success rate** is $\pi^*(3) = 3/6 = 50.00\%$.

Rank order	1, 2, <u>3</u>	1, 3, <u>2</u>	2, <u>1</u> , 3	2, 3, <u>1</u>	3, <u>1</u> , 2	3, <u>2</u> , 1
Pick?	3rd	3rd	2nd	3rd	2nd	2nd
Win?	No	No	Yes	Yes	Yes	No

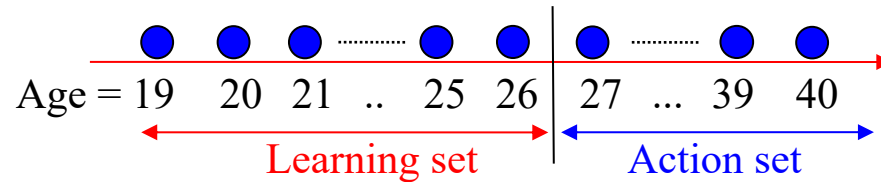
- If $n \rightarrow \infty$, the size of the learning set $L \rightarrow n/e = 0.368 n$ and the success rate is $\pi \rightarrow 1/e = 36.8\%$, where $e = 2.71828$

Excel #2



* Lindley's (1961) Marriage Problem

- "If, in real life, this process works between 18 and 40 (i.e., for 22 years), one should **never** propose until age 26."



Why I'm always **happy**,
but my wife is **not**?

- He explained, "Either many people do **not** pursue an **optimal strategy** or else they have a different **utility function**."
- In real life, Do we have **one** candidate a year?
Are you looking for nothing but the **best**?
It is a mating **game** between males and females.
Your marriage proposal may be **rejected** 😞

My "**tried and true**" advice (for **free**!)

"There are two approaches to love - through the **heart** and through the **head**. Neither one seems to work very well alone but, **together**, there's probably a better chance of **success**..."

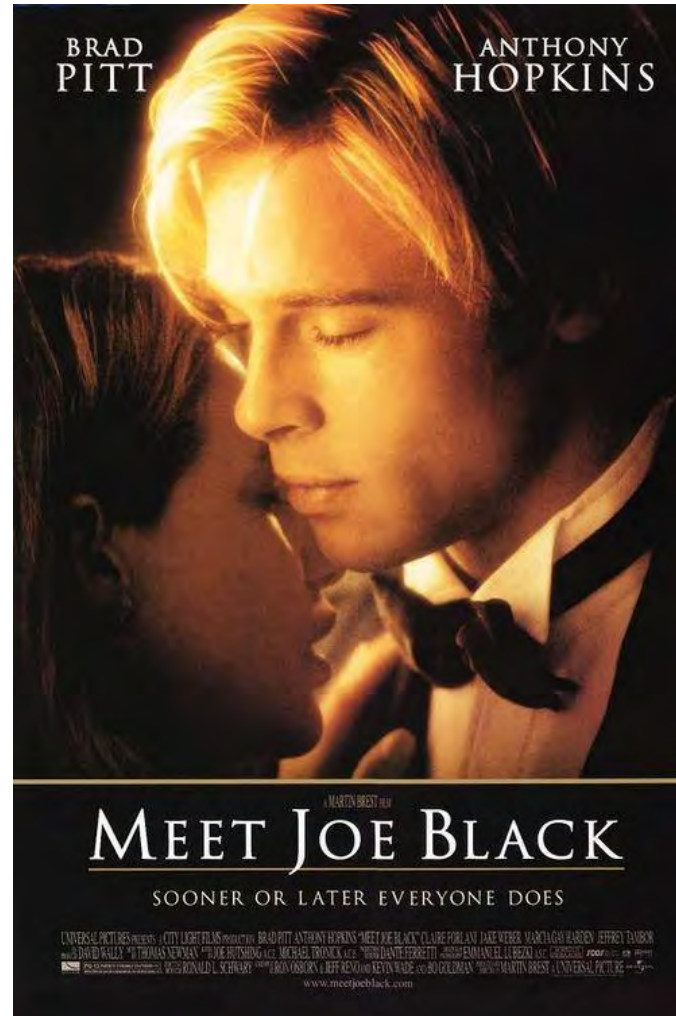
Movie Trivia



A media mogul acts as a guide to **Death**, who takes the form of a **young man** to learn about life on Earth and, in the process, falls in love with his guide's **daughter**.



Meet Joe Black (1998)



A media mogul acts as a guide to **Death**, who takes the form of a **young man** to learn about life on Earth and, in the process, falls in love with his guide's **daughter**.



* Memorable Quotes?

■ Dad:

"**Love** is passion, obsession, someone you can't live without. I say, fall head over heels. Find someone you can love like crazy and who will love you the same way back.



How do you find him? Well, you forget your **head**, and you listen to your **heart**. Because the truth is, honey, there's no sense living your **life** without this. To make the journey and not fall deeply in love, well, you haven't lived a **life** at all. But you have to try, because if you haven't tried, you haven't lived."

■ His Daughter:



"**Love, passion, obsession**, all those things you told me to wait for? Well, they've arrived. What are you afraid of, Dad? That I'll fall head over heels for **Joe**? Well, **I have...**"