LSU E. J. OURSO COLLEGE OF BUSINESS

# Dr. Chun's Numb3rs & L∅gic

## *Introduction to Game Theory*

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# Game Theory

Game theory attempts to mathematically capture behavior in strategic situations, in which an individual's success in making choices depends on the choices of others.

**1. Two-person zero-sum game:**

LSU has the ball…

| | | Nick Savan | |
|---|---|---|---|
| | | $b_1$: Run | $b_2$ Pass |
| Les Miles | $a_1$: Run | -3 yards | +4 yards |
| | $a_2$: Pass | +12 yards | -8 yards |

- The *optimal* strategy for each coach? Pure or mixed?
- Is this a fair game?
- The *value* of the game? Average gain or loss?

## 2. Two-person general-sum game:

What to wear to the TGIF party?

| | | Your Friend | |
|---|---|---|---|
| | | $b_1$: Wears the same shirt | $b_2$: Doesn't wear the shirt |
| You | $a_1$: Wear your favorite shirt | You both look like copycats. **(-10, -10)** | You look cool, your friend doesn't. **(8, -2)** |
| | $a_2$: Don't wear your favorite shirt | Your friend looks cool, but you don't. **(-2, 8)** | Neither of you look cool or dorky. **(0, 0)** |

- The *optimal* strategy for each player?

## 3. *n*-person zero-sum game

# Prisoner's Dilemma

A taste of the Game Theory!

The so-called prisoner's dilemma owes its name to the scenario wherein two men suspected of a burglary are arrested in the course of committing some minor offense. They're separated and interrogated, and each is given the choice of confessing to the burglary and implicating his partner, or remaining silent.

If they both remain silent, they'll each receive only one year in prison. If one confesses and the other doesn't, the one who confesses will be rewarded by being let go, while the other one will receive a ten-year term. If they both confess, they can both expect to spend five years in prison. The cooperative option is thus to remain silent, while the individualist option is to confess.

The dilemma is that what's best for them as a pair, to remain silent and spend a year in prison, leaves each of them open to the worst possibility, being a patsy and spending ten years in prison. As a result they'll probably both confess and both spend five years in prison.

## * Loss Table

| Loss | | Partner | |
|---|---|---|---|
| | | $s_1$: Confess | $s_2$: Deny |
| You | $d_1$: Confess | { **5**, 5 } | { **0**, 10 } |
| | $d_2$: Deny | { **10**, 0 } | { **1**, 1 } |

## * Decision Making without Probabilities

| Loss | $s_1$ | $s_2$ | LaPlace | Minmax | Minimin | Minimax Regret |
|---|---|---|---|---|---|---|
| $d_1$ | 5 | 0 | 2.5 | 5 | 0 | 0 |
| $d_2$ | 10 | 1 | 5.5 | 10 | 1 | 5 |
| | | | $d_1$ | $d_1$ | $d_1$ | $d_1$ |

- The optimal decision is $d_1$ (Confess) unanimously!

## * Dominance Relationship?

$d_1$ (Confess) *dominates* $d_2$ (Deny)

## * Pure strategy

| Loss | | Partner | |
|---|---|---|---|
| | | $s_1$: Confess | $s_2$: Deny |
| You | $d_1$: Confess | { **5**, 5 } | { **0**, 10 } |
| | $d_2$: Deny | { **10**, 0 } | { **1**, 1 } |

- Local and global minimum points
  - Individual vs. group interest
  - NIMBY? Not in my back yard…
    Nuclear power plant, maximum security prison, …
- "Nash equilibrium" point for strategic non-cooperative games
  - Dr. John Nash in "*A Beautiful Mind (2001)*"
    Father of game theory; 1994 Nobel prize winner…

# His college professor wrote a letter of recommendation consisting of a *single* sentence: "*This man is a genius.*"

# Think *twice* before you ask me for a letter of recommendation? I'm going to write one sentence, "***This man is an idiot.***"

# Rock, Paper, Scissors

Each of two players simultaneously utters one of the three words: rock, paper, or scissors. If both players utter the same word, the game is a draw. Otherwise, one player wins or loses $1 from the other according to the following payoff table:

| Your payoff | | Your friend | | |
|---|---|---|---|---|
| | | $b_1$: Rock | $b_2$: Paper | $b_3$: Scissors |
| You | $a_1$: Rock | 0 | -1 | +1 |
| | $a_2$: Paper | +1 | 0 | -1 |
| | $a_3$: Scissors | -1 | +1 | 0 |

- The optimal mixed strategies are ( **1/3**, **1/3**, **1/3** ).
- The value of the game is **$0** (i.e., fair game)

# Ask Marilyn, Parade Magazine (March 31, 2002), page 18

"Let's show pennies to each other, either heads or tails.
If we both show heads, I pay you $3.
If we both show tails, I pay you $1.
And if they don't match, you pay me $2."

| My payoff | | Student | |
|---|---|---|---|
| | | $b_1$: Head | $b_2$ Tail |
| Professor | $a_1$: Head | -3 | +2 |
| | $a_2$: Tail | +2 | -1 |

- The *optimal* mixed strategy for the professor?
  Head:Tail = 3:5

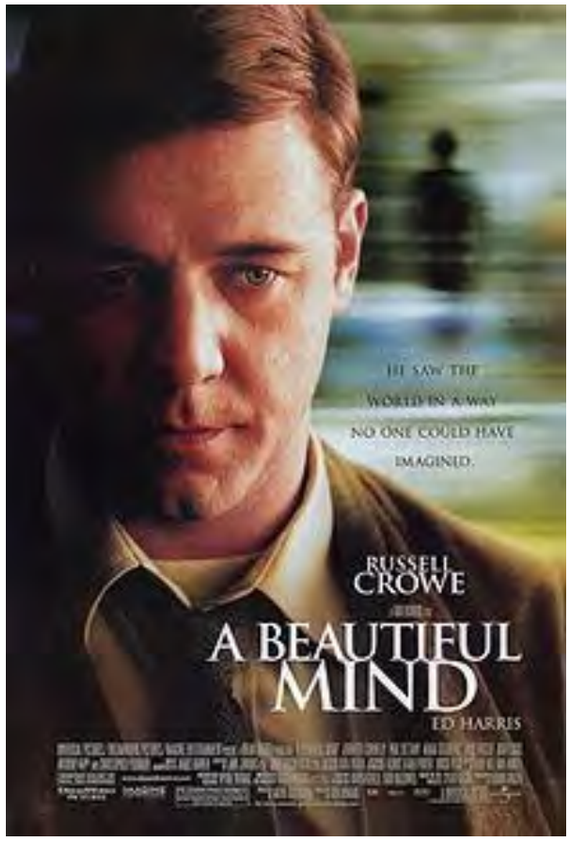
- The *value* of the game?
  +$1/8

# Movie Trivia


Biopic of the famed mathematician John Nash and his lifelong struggles with his mental health.

After the brilliant but asocial mathematician accepts secret work in cryptography, his life takes a turn to the nightmarish.

# A Beautiful Mind (2001)

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