

Dr. Chun's Numb3rs & Løgic

Nash Equilibrium



Young H. Chun, Ph.D.

*Professor of Decision Science &
Cherie H. Flores Endowed Chair in MBA Studies*



The Traveler's Dilemma

Lucy and Pete, returning from a remote Pacific island, find that the airline has damaged the **identical antiques** that each had purchased. An **airline manager** says that he is happy to compensate them but is handicapped by being clueless about the value of these strange objects.



He asks each of them to write down the price of the **antique** as any dollar integer between **2** and **100** without conferring together. If both write the **same number**, he will take that to be the **true price**, and he will pay each of them that amount.

But if they write **different numbers**, he will assume that the **lower one** is the actual price and that the person writing the **higher number** is cheating. In that case, he will pay both of them the **lower number** along with a **bonus** and a **penalty**



-- the person who wrote the *lower* number will get \$2 more as a **reward** for honesty and the one who wrote the *higher* number will get \$2 less as a **punishment**. For instance, if **Lucy** writes 46 and **Pete** writes 100, **Lucy** will get \$48 and Pete will get \$44.

What numbers will **Lucy** and **Pete** write? What number would you write?



* Payoff Table

Payoff		Pete						
		100	99	98	..	4	3	2
Lucy	100	(100, 100)	(97, 101)	(96, 100)	..	(2, 6)	(1, 5)	(0, 4)
	99	(101, 97)	(99, 99)	(96, 100)	..	(2, 6)	(1, 5)	(0, 4)
	98	(100, 98)	(100, 98)	(98, 98)	..	(2, 6)	(1, 5)	(0, 4)

	4	(6, 2)	(6, 2)	(6, 2)	..	(4, 4)	(1, 5)	(0, 4)
	3	(5, 1)	(5, 1)	(5, 1)	..	(5, 1)	(3, 3)	(0, 4)
	2	(4, 0)	(4, 0)	(4, 0)	..	(4, 0)	(4, 0)	(2, 2)

- Can you find the **Nash equilibrium** point?





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- Can you find the **Nash equilibrium point**?
- It has only one **Nash equilibrium**, ?





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- Can you find the Nash equilibrium point?
- It has only one Nash equilibrium, (2, 2). Make sense to you?





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		99	
Lucy	100		(97, 101)
	99		(99, 99)
	98		(100, 98)

	4		(6, 2)
	3		(5, 1)
	2		(4, 0)

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	98							
	..							
	4							
	3							(0, 4)
	2						(4, 0)	(2, 2)

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- Can you find the Nash equilibrium point?
- It has only one Nash equilibrium, (2, 2). Make sense to you?
- **Game theory** is a **valid model** or a **myth**?





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- Can you find the Nash equilibrium point?
- It has only one Nash equilibrium, **(2, 2)**
- Game theory is a valid model or a myth?
- Improve the **theory** and you could win the **next Nobel prize!**





An Inconvenient Puzzle

John Allen Paulos, *Who's counting*, ABC News (May 7, 2007)

"The Imp in the Bottle" is the story of a **genie** in a bottle who will satisfy your every wish for **love**, **money**, and **power**. You can buy this amazing bottle for any amount that you care to offer.



The only **constraint** is that when you are finished with the bottle, you must sell it for a price strictly **less than** that you paid for it.

If you don't sell it to someone for a lower price, you will lose everything and suffer everlasting torment in hell. (**Everyone** knows that it must be sold for a price less than the price at which it is bought.)

What would you pay for such a **magic bottle**?



Certainly, you won't pay **1 cent** for it because then you won't be able to sell it for a lower price. You won't pay **2 cents** for it either because no one will buy it from you for 1 cent for the same reason.

Neither will you pay **3 cents** for it; the person to whom you would have to sell it for **2 cents** would object to buying it at that price since he wouldn't be able to sell for **1 cent**.

A similar argument applies to a price of **4 cents**, **5 cents**, **6 cents** and so on. **Mathematical induction** can be used to formalize this argument, which proves conclusively that you shouldn't buy this **magic bottle** for any amount of money.

Yet you would almost certainly buy it for **\$1,000**. I know I would. At what point does the argument against buying the bottle become practically convincing?

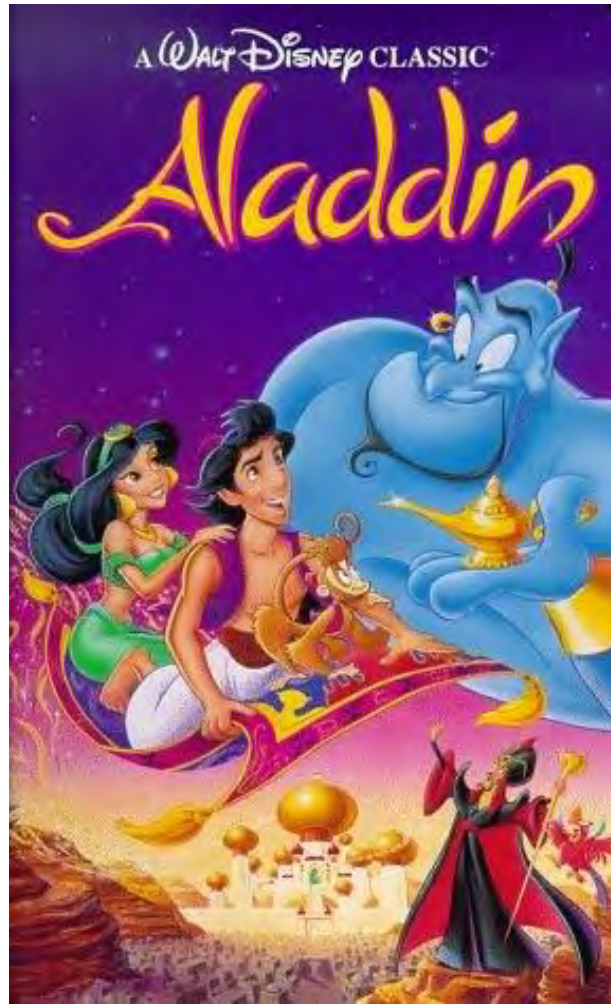


Movie Trivia



A street urchin, accidentally meets **Princess Jasmine**,
who is in the city undercover.
They love each other, but she can only marry a **prince**.

Aladdin (1992)



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