LSU E. J. OURSO COLLEGE OF BUSINESS

Dr. Chun's Numb3rs & L∅gic

St. Petersburg Paradox

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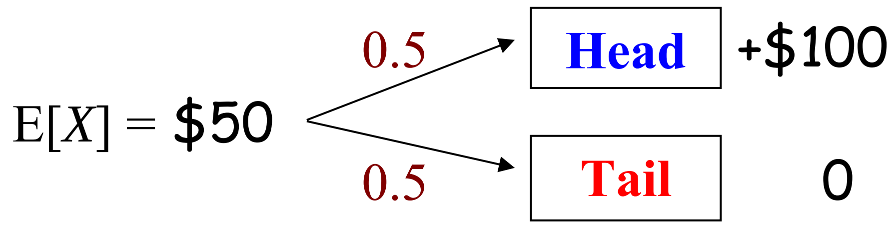
1. Game for Standard Household Doorknobs

I will flip a fair coin. If it lands heads, you will receive $100. If it lands tail, you will receive $0.

How much would you be willing to pay to play this game?



- Are you willing to pay me $70? No way!
- Are you willing to pay me $30? Of course!
- The expected value of the game is $50!

> You want to play the game if
> Fee < Value of the game (=$50)

2. Game for Rocket Scientists

From "Ask Marilyn," *Parade Magazine*, (August 3, 1997), p. 14

"Suppose I offered to play a certain game with you for a fee:

I would flip a fair coin, and it landed heads, I would give you $2 – game over.

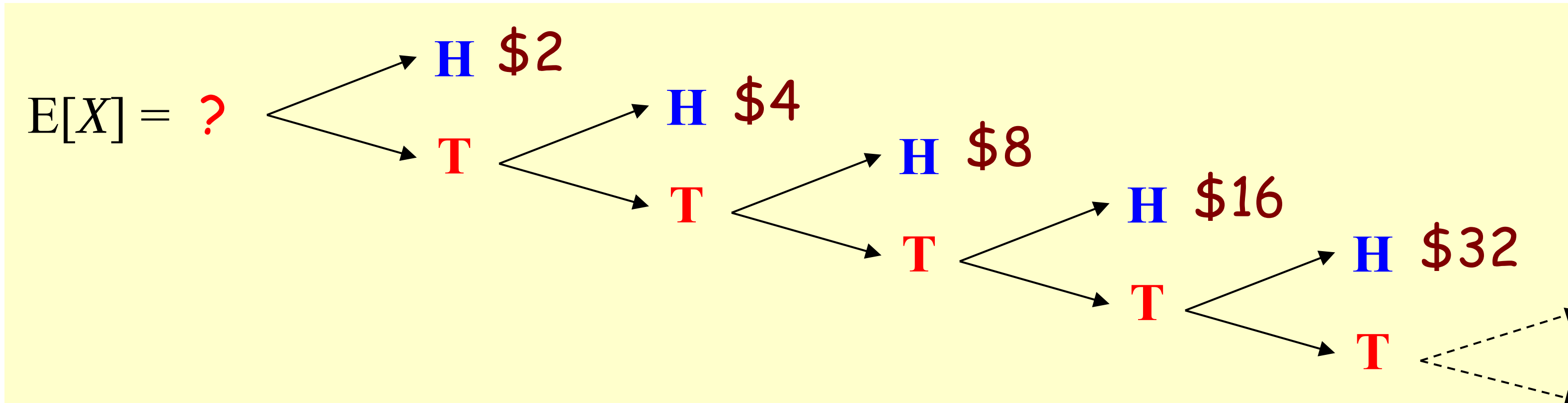
If it landed tails, I would flip it again. If, on the second try, it landed heads, I would give you $4 – game over. If tails, I'd flip again.

Heads on the third flip, I'd pay $8, and so on ($16, $32, $64,...). I'd keep flipping until heads came up, at which time I'd pay you – game over.

How much would you be willing to pay to play this game?"

– Eric Dobkin, Watchung, N.J.

- Tree diagram



- Expected value of the game? **Infinity!**

	1	2	3	4	…	i	…	∞
Payoff, $\$X_i$	**2**	**4**	**8**	**16**	**…**	$\mathbf{2^i}$	**…**	**…**
Probability, P[i]	**1/2**	**1/4**	**1/8**	**1/16**	**…**	$\mathbf{(1/2)^i}$	**…**	**…**
$P[i]\,X_i$	**1**	**1**	**1**	**1**		**1**	**…**	**1**

$$E[X] = \sum_{i=1}^{\infty} P[i]\, X_i = \infty$$

Contestants, Taxes, Paradoxes and Sure Things

John Allen Paulos, Who's counting, ABC News (Oct.2, 2005)

The St. Petersburg paradox usually takes the form of a game requiring that you flip a coin repeatedly until a head first appears.

If a head appears on the first flip, you win $2. If the first head appears on the second flip, you win $4. If the first head appears on the third flip, you win $8, and, in general, if the first head appears on the Nth flip, you win 2^N dollars.

It can be shown that you should be willing to pay any price to play this game. No matter how much you bet each time you play, you'll still come out way ahead on average.

The notion of utility resolves the famous St. Petersburg paradox.

St. Petersburg Paradox

You want to play the game if
Fee < Value of the game (=$infinity)

- Do you want to pay one million dollars to play the game?
- Why most people do not want to pay more than $10?
- How to resolve this paradox?

*** Risk preference in decision analysis**

1. Risk-averse: Most people are risk averse...
2. Risk-neutral Expected payoff
3. Risk-prone: Lottery ticket

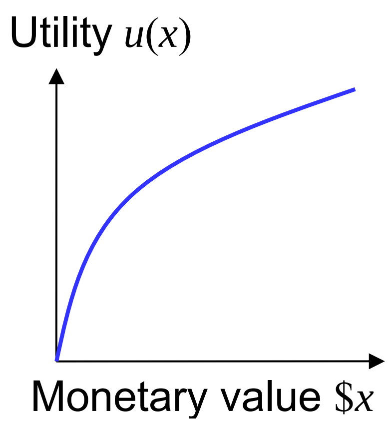
*** Utility Theory:**

Utility is a function of the payoff...

Utility Theory

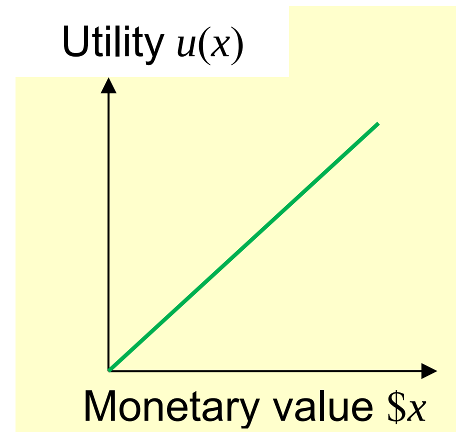
- **Different types of utility function**

Concave function



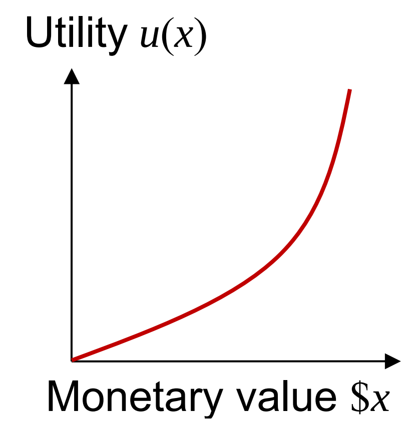
Risk-averse

Linear function



Risk-neutral

Convex function



Risk-prone

If we assume that we are risk-neutral, then

E[Payoff] is equivalent to E[Utility] !

* Risk averse with utility function $u(x) = \sqrt{x}$

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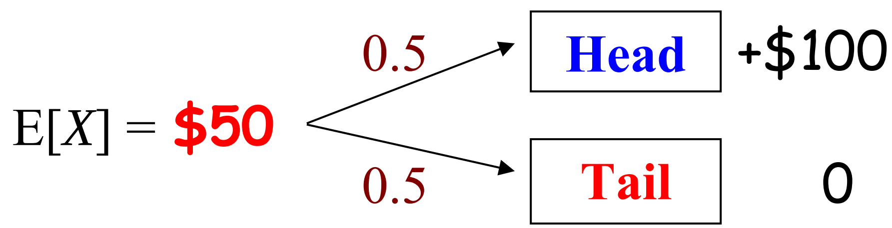


- Expected value is E[X] = \$50
- Value of the game is only \$25!

* Risk-prone? Lottery ticket!

Expected value = 50 cents < Ticket price = \$1.00

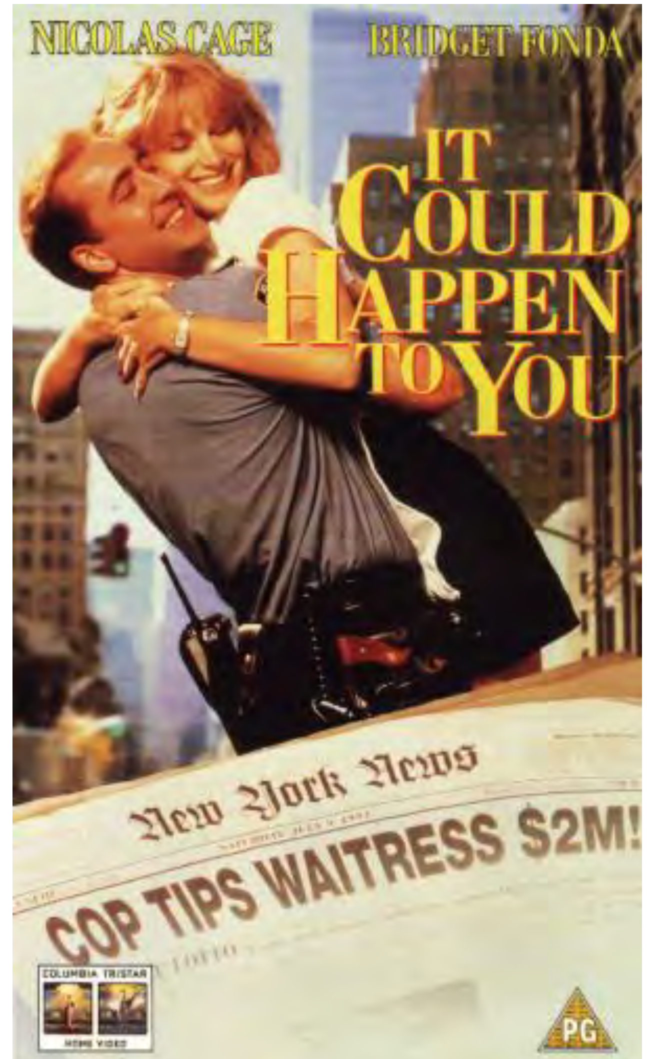
"*The lottery is a tax on people who flunked math.*"

– Monique Lloyd

Movie Trivia

A police officer promises to share his lottery ticket with a waitress in lieu of a tip.

It Could Happen to You (1994)



A police officer promises to share his lottery ticket with a waitress in lieu of a tip.