LSU E. J. OURSO COLLEGE OF BUSINESS

Dr. Chun's Numb3rs & L∅gic

70! Problem and Optimization

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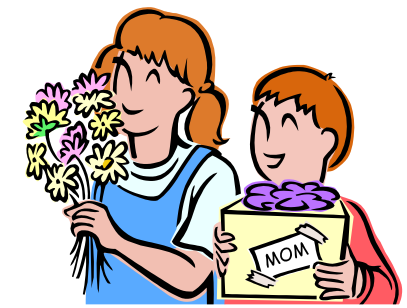
Gift Exchange

From “Ask Marilyn,” *Parade Magazine*, (September 13, 1992), p. 26

"After each holiday gift exchange, my five nieces and nephews write their names on slips of paper into a basket, from which they then draw the name of the person for whom they'll buy a gift the following year.



Last year, for the first time, each of the five drew his or her own name. What are the chances of such an occurrence taking place?

– Richard Coffey, Newington, Conn."

Adam	Brett	Cindy	David	Emily

* Assignment problem with 5 kids

	1 Adam	2 Brett	3 Cindy	4 David	5 Emily
1. Adam	0				
2. Brett		0			
3. Cindy			0		
4. David				0	
5 .Emily					0

* **Total number of possible assignments** (i.e., permutations)?

5! = 120

* Probability of the perfect match?

1/5! = 1/120

Adam	Brett	Cindy	David	Emily

$$\frac{1}{5}\ \frac{1}{4}\ \frac{1}{3}\ \frac{1}{2}\ \frac{1}{1} = 1/5! = 1/120$$

* Assignment problem with 70 jobs and 70 machines

Completion time		Machine				
		1	2	3	…	70
Job	1	7	2	3	.	1
	2	4	2	2	.	5
	3	5	4	0	.	2
	…	.	.	.	.	.
	70	3	2	4	.	6

* Total number of possible assignments (i.e., permutations)?

70! = ?

* How to find the optimal solution?

- Paper and pencil?
- Computer software!
  Linear Programming (LP)

70! Problem

* **Statement 1** by Professor G. B. Dantzig (father of LP) at Stanford

"Now 70! is a big number, greater than 10^{100}. Suppose we had an IBM 370-168 available at the time of the Big Bang 15 billion years ago. Would it have been able to look at all 70! solutions by 1981? No.

Suppose instead it could examine one billion assignments per second? The answer is still no. Even if the Earth were filled with such computers all working in parallel, the answer would still be no."

From "Reminiscences about the origins of linear programming," *Mathematical Programming*, 6, 1984, pp. 105-112.

* **Statement 2** by Professor Chun (father of two teen-age kids) at LSU

"Given an assignment problem with 70 machines and 70 jobs, we can find the most efficient assignment in *several seconds* using Microsoft Excel, for example."

* Whose side are you on?

* LP Formulation with n=3

Time a_{ij}		Machine 1	Machine 2	Machine 3
Job	1	4	1	3
	2	2	3	6
	3	4	8	9

- Decision variables: $x_{ij} = \begin{cases} 1 & \text{if } i \text{ is assigned to } j \\ 0 & \text{if not} \end{cases}$

		Machine 1	Machine 2	Machine 3
Job	1	X_{11}	X_{12}	X_{13}
	2	X_{21}	X_{22}	X_{23}
	3	X_{31}	X_{32}	X_{33}

- Any heuristic algorithm? *Nearest Neighbor*: Smallest first!

$1X_{12}$ => $2X_{21}$ => $9X_{33}$ The total cost is **12** hours

$3X_{13}$ => $3X_{22}$ => $4X_{31}$ The total cost is **10 hours**

* LP Formulation with n=3

Time a_{ij}		Machine 1	Machine 2	Machine 3
Job	1	4	1	3
	2	2	3	6
	3	4	8	9

- Decision variables:

$$x_{ij} = \begin{cases} 1 & \text{if } i \text{ is assigned to } j \\ 0 & \text{if not} \end{cases}$$

- Objective function

$$\text{Min } z = 4x_{11} + 1x_{12} + 3x_{13} + 2x_{21} + 3x_{22} + 6x_{23} + 4x_{31} + 8x_{32} + 9x_{33}$$

- Constraints

$x_{11} + x_{12} + x_{13} = 1$

$x_{21} + x_{22} + x_{23} = 1$

$x_{31} + x_{32} + x_{33} = 1$

$x_{11} + x_{21} + x_{31} = 1$

$x_{12} + x_{22} + x_{32} = 1$

$x_{13} + x_{23} + x_{33} = 1$

* LP Formulation with *n*

Time a_{ij}		Machine		
		1	2	*n*
Job	1	X_{11}	X_{12}	X_{1n}
	2	X_{21}	X_{22}	X_{2n}
	n	X_{31}	X_{32}	X_{nn}

- Decision variables: $$x_{ij} = \begin{cases} 1 & \text{if } i \text{ is assigned to } j \\ 0 & \text{if not} \end{cases}$$

- Objective function $$\min\ z = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_{ij}$$

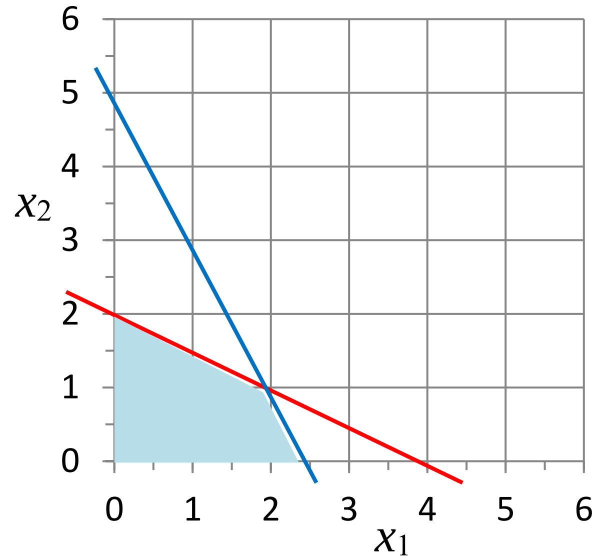
- **Constraints** $$\sum_{i=1}^{n} x_{ij} = 1 \qquad \text{for each } j=1,2,\ldots,n$$

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad \text{for each } i=1,2,\ldots,n$$

* **Feasible region**

- A set of *solutions* that satisfy all the *constraints*.

- 2 variables



- Corner points

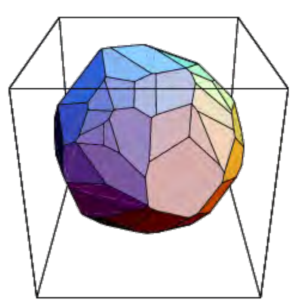
(x_1, x_2)	z
(0, 0)	**0**
(2.5, 0)	**2.5**
(2, 1)	**3**
(0, 2)	**2**

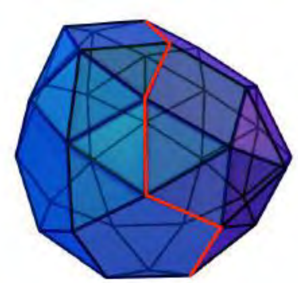
- Optimal solution:

$x_1 = 2$ and $x_2 = 1$

with $z^* = 3$

- 3 variables

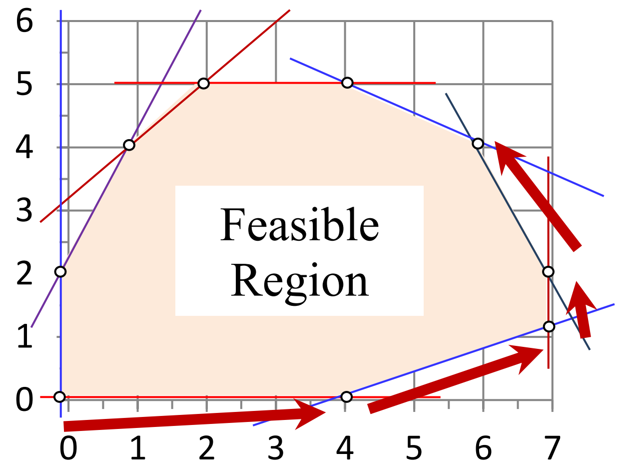




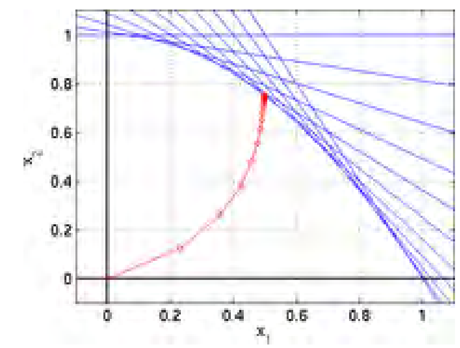
- 4 variables?

?

Ex] Simplex method



Ex] Interior point method



- Objective function: max $z = x_1 + x_2$

(x_1, x_2)	(0, 0)	(4, 0)	(7, 1)	(7, 2)	(6, 4)	(4, 5)	(2, 5)	(1, 4)	(0, 2)
z	0 →	4 →	8 →	9 →	10 →	9 ←	7	5	2

- Simplex method

  Developed by George Dantzig at Stanford in 1947.

- Interior point method

  Developed by Narendra Karmarkar in 1984.

* **Traveling Salesman Problem** (TSP):

"Given a number of cities and the costs of traveling from any city to any other city, what is the cheapest round-trip route that visits each city exactly once and then returns to the starting city?"

* **Heuristic algorithms**: *Nearest neighbor* algorithm:

Easy to implement and executes quickly.

* **Optimal solution algorithms**

- Solved for visiting 42 cities in USA in 1954.
- Solved for visiting 120 cities in West Germany in 1977.
- Solved for visiting all 532 switch location in USA in 1984.
- Solved for visiting 15,112 cities in Germany in 2001.
- Solved for visiting all 24,978 cities in Sweden in 2004.
- Solved for visiting all 33,810 points in a circuit board in 2005.

- $1 million for visiting any number of points!