

Dr. Chun's Numb3rs & Løgic

Fermat's Last Theorem Analysis



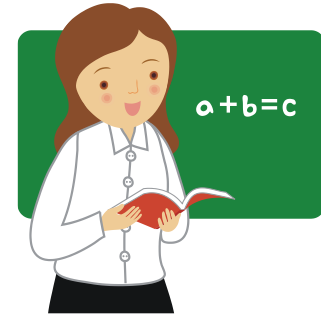
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Diophantine Analysis

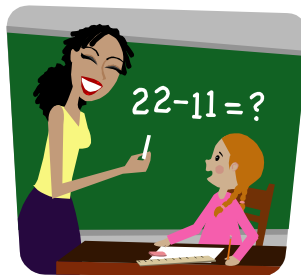


In mathematics, a **Diophantine equation** is an equation in which only **integer solutions** are allowed. Diophantine problems have **fewer equations** than **unknown variables** and involve finding **integers** that work correctly for all **equations**.



In more technical language, they define an algebraic **line**, algebraic **curve**, algebraic **surface**, or more **general object**, and ask about the **lattice points** on it.

The word **Diophantine** refers to the Hellenistic mathematician of the 3rd century, **Diophantus** of **Alexandria**, who made a study of such equations and was one of the first mathematicians to introduce **symbolism** into algebra.





One Equation with Two Variables

A father's age is 1 less than twice that of his son, and the digits XY making up the father's age are reversed in the son's age (i.e., YX). How old is the father?

* Ages

- Father's age = $10X + Y$
- His son's age = $10Y + X$

* Equation:

$$10X + Y = 2 \times (10Y + X) - 1 = 20Y + 2X - 1$$

$$8X = 19Y - 1 \quad \text{or} \quad Y = (8/19)X + 1/19$$



* Dilemma

- We have only one equation, but two unknown variables!



* Diophantine equation:

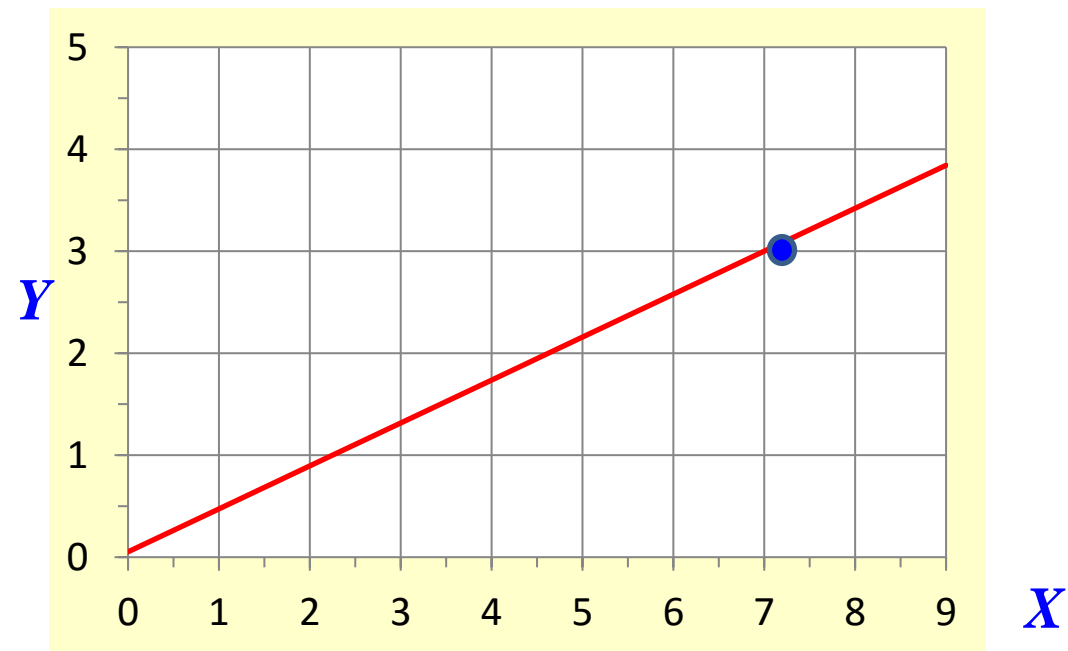
- $Y = (8/19)X + 1/19$

where X and Y are **integers** in the range of 0 and 9.

- You may find the unique solution by a **complete enumeration!**

* Solution

X	Y
0	0.053
1	0.474
2	0.895
3	1.316
4	1.737
5	2.158
6	2.579
7	3.000
8	3.421
9	3.842



- Only one pair of **integer values** on the **algebraic line!**
 - Thus, the **father's age is 73**, and the **son's age is 37!**

Fermat's Last Theorem



"No three **positive integers** a , b , and c can satisfy the equation $a^n + b^n = c^n$ for any integer value of n greater than 2."

And then **Pierre de Fermat** wrote in 1637, intriguingly:

"I have discovered a truly marvelous **proof** of this, which, however, the **margin** is not large enough to contain."



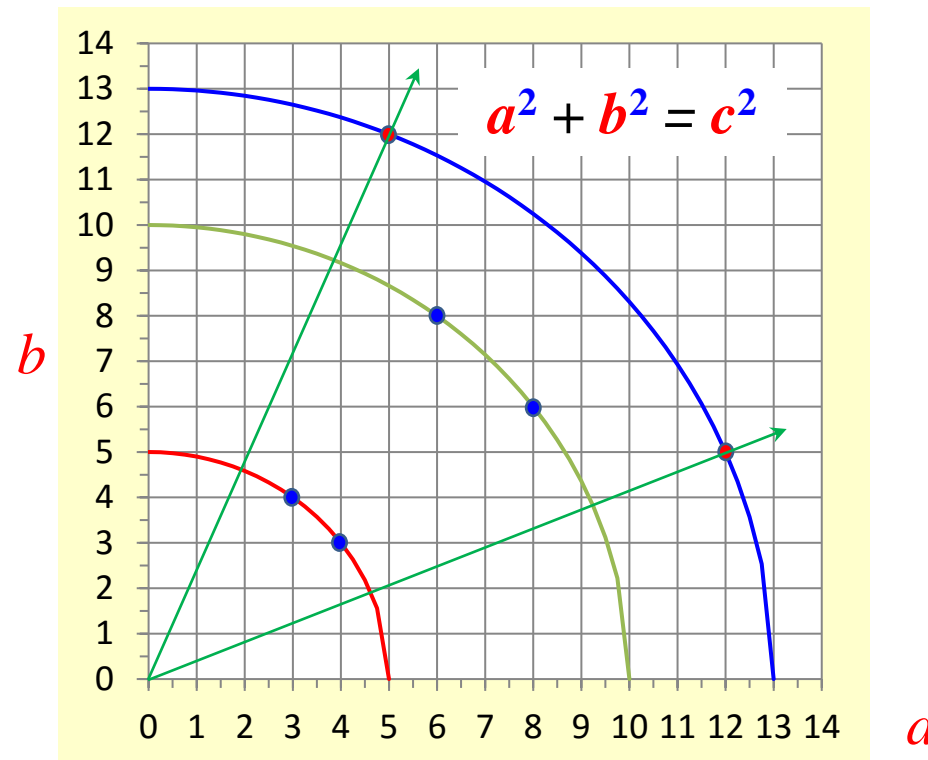
Such a proof eluded **mathematicians** for centuries, however. As an unproven **conjecture** that eluded brilliant mathematicians' attempts to either prove it or disprove it for generations, his statement became famous as **Fermat's Last Theorem**.

In fact, **one million dollars** is waiting to be won by anyone who can solve one of many grand mathematical challenges of the 21st century.



* Pythagorean triples: $a^2 + b^2 = c^2$

- There are a set of 3 integers (e.g., 3, 4, 5 or 5, 12, 13) that satisfy a special case of Fermat's equation with $n=2$.



a	3	6	5	9	10	...
b	4	8	12	12	24	...
c	5	10	13	15	26	...



* Fermat's last theorem:

- The equation $a^n + b^n = c^n$ has no solution if a , b , and c are positive integers and if n is an integer greater than 2.

$$a^3 + b^3 = c^3$$

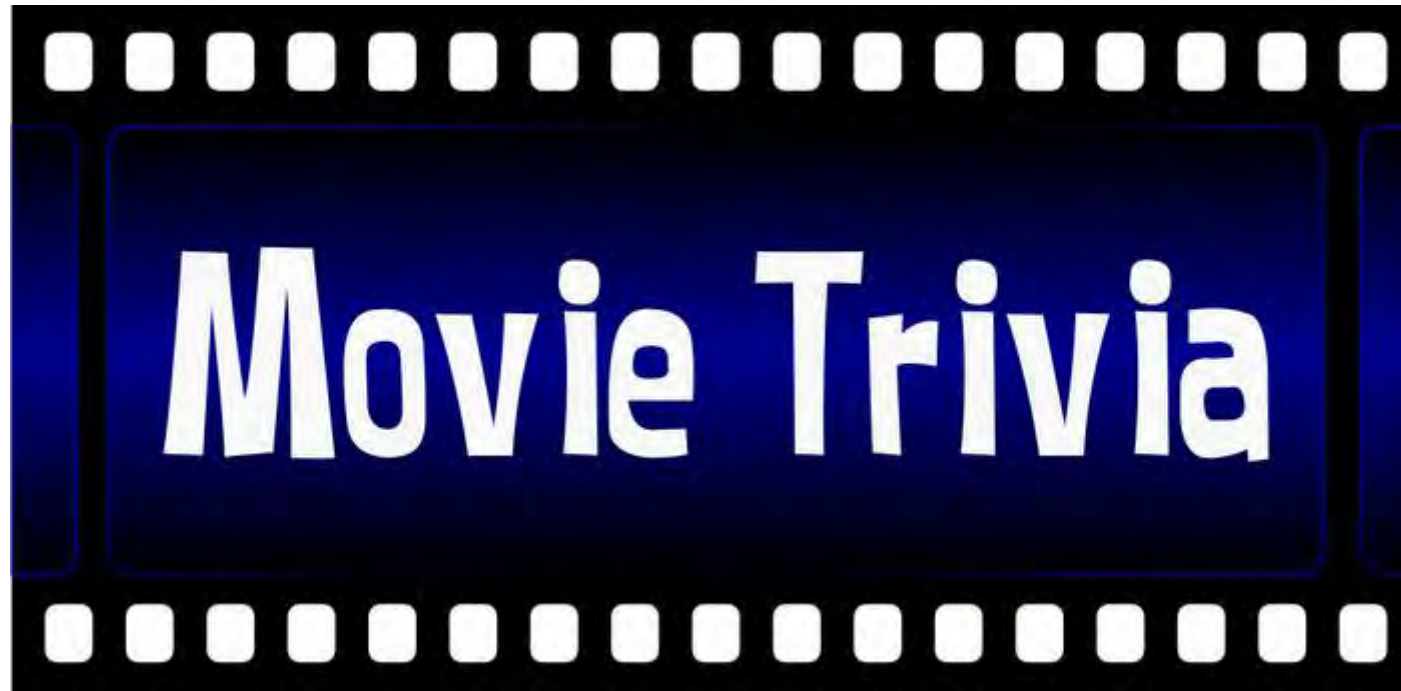
$$a^4 + b^4 = c^4$$

$$\dots \dots = \dots$$

- The **conjecture** was proved for some **special cases** where $n = 3$, 5, and 7.
- It is among the **most famous theorems** in the history of mathematics and was in the *Guinness Book of World Records* for "the most difficult mathematical problems".
- The theorem was finally proved by a **British mathematician Andrew Wiles** in **1994**, **358 years** after it was conjectured!

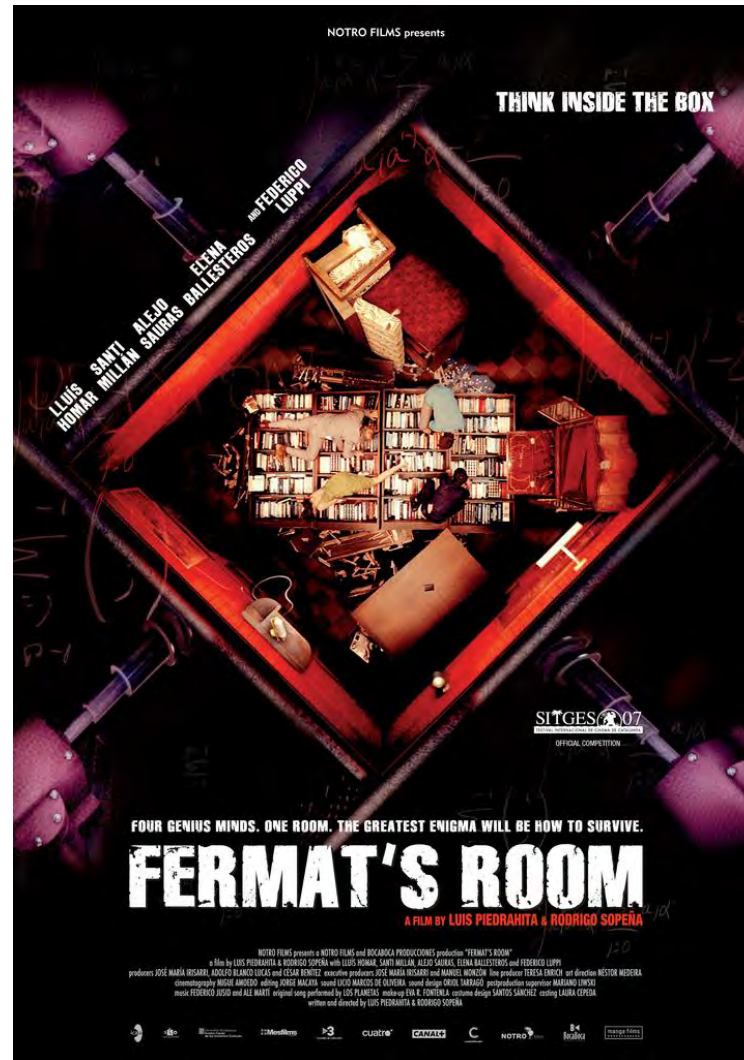


Movie Trivia



Four **mathematicians** who do not know each other are invited by a mysterious host on the pretext of resolving a great **enigma**.

Fermat's Room (2007)



Four **mathematicians** who do not know each other are invited by a mysterious host on the pretext of resolving a great **enigma**.