

Dr. Chun's Numb3rs & Løgic

Broken Stick



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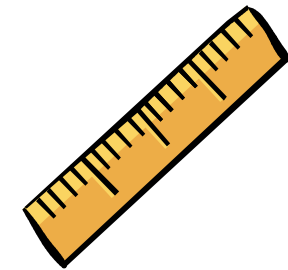


Broken Stick - Part I

From *Introduction to Probability*, by Charles Grinstead and J. Laurie Snell

Take a **stick** of unit length and break it into **two pieces**, choosing the break point at random.

What is the **average length** of the **shorter piece**?



▪ Variable

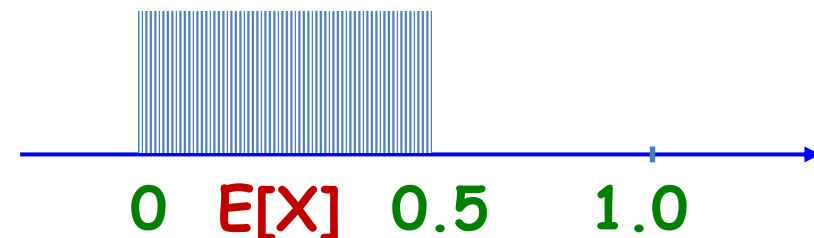
X = length of the **shorter** piece

$1-X$ = length of the longer piece

▪ Average length of the **shorter** piece

$$\begin{aligned} E[X \mid X < 1-X] &= E[X \mid X < 0.5] \\ &= \mathbf{0.25} \end{aligned}$$

Uniform
distribution



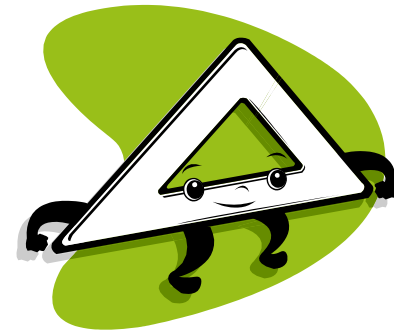
Broken Stick - Part II



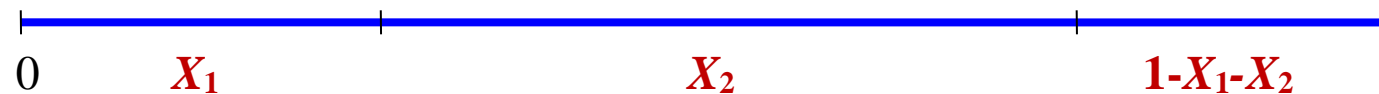
From *Introduction to Probability*, by Charles Grinstead and J. Laurie Snell

Take a **stick** of unit length and break it into **three pieces**, choosing the break points at random. (The break points are assumed to be chosen simultaneously.)

What is the **probability** that the three pieces can be used to form a **triangle**?



Condition: The longest piece among the three should be less than 0.5.



X_1 = Length of the **first** piece

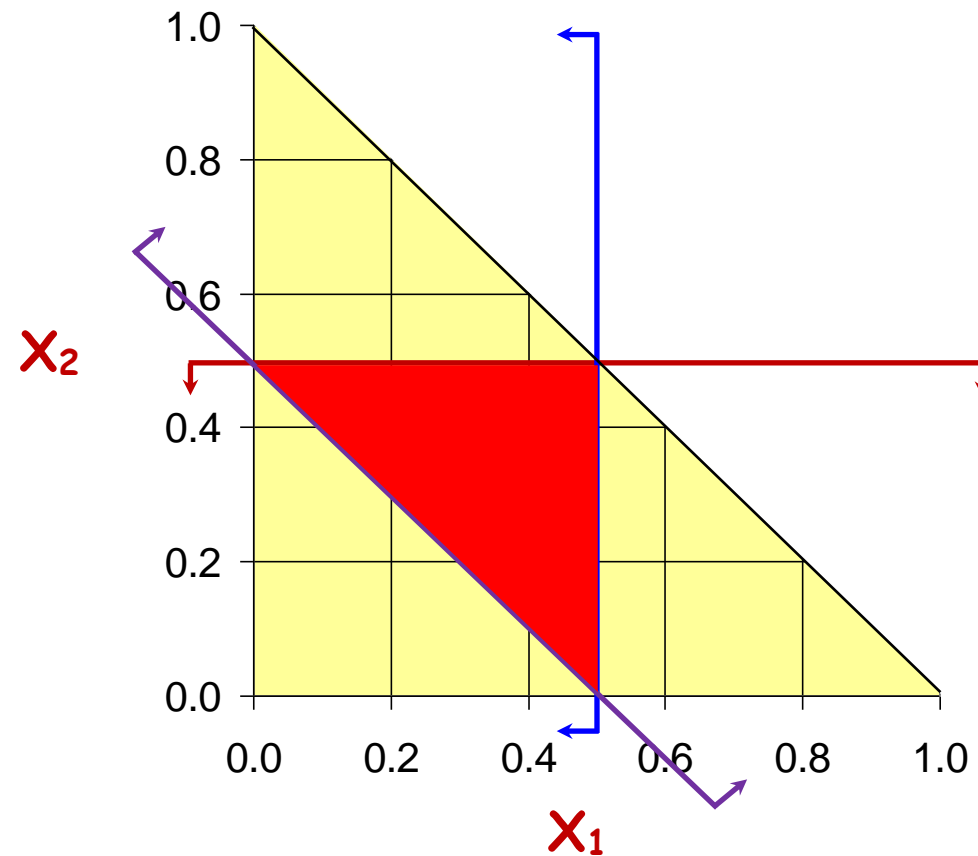
X_2 = Length of the **second** piece, where $X_1+X_2 < 1$

$1-X_1-X_2$ = Length of the **third** piece



- **Conditions:** The longest piece should be less than 0.5

$$X_1 < 0.5, \quad X_2 < 0.5, \quad 1 - X_1 - X_2 < 0.5 \quad \text{or} \quad X_1 + X_2 > 0.5$$



The area of the red triangle? $1/4 = 25\%$

Movie Trivia



When a woman's long-time friend says he's engaged, she realizes she loves him herself... and sets out to get him, with only a few days before the wedding.

My Best Friend's Wedding (1997)



When a woman's long-time friend says he's engaged, she realizes she loves him herself... and sets out to get him, with only a few days before the wedding.